Quantum Systems, Information, and Entanglement

Lecture 1. Introduction to Quantum Systems

Spyros Tserkis

Postdoctoral Fellow

April 13, 2022

Center for Quantum Networks

HARVARD UNIVERSITY

NARANGLAB MATERIALS THEORY AT HARVARD

Center for Quantum Networks

There are four main thrusts to CQN:

Thrust 1: Quantum network architecture

Thrust 2: Quantum sub-system technologies

Thrust 3: Quantum materials, devices and fundamentals

Thrust 4: Societal impact of the Quantum Internet

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Course Outline

- **Lecture 1 — Introduction to Quantum Systems** (April 13, 2022)
- **Lecture 2 — Teleportation and Entanglement** (April 20, 2022)
- **Lecture 3 — Decoherence and Quantum Networks** (April 27, 2022)

Lecture 1 — Introduction to Quantum Systems

- Introduction
  - When Classical Mechanics Fails
  - Review of Classical Systems
- Quantum Observables
- Quantum States
- Evolution in Quantum Systems
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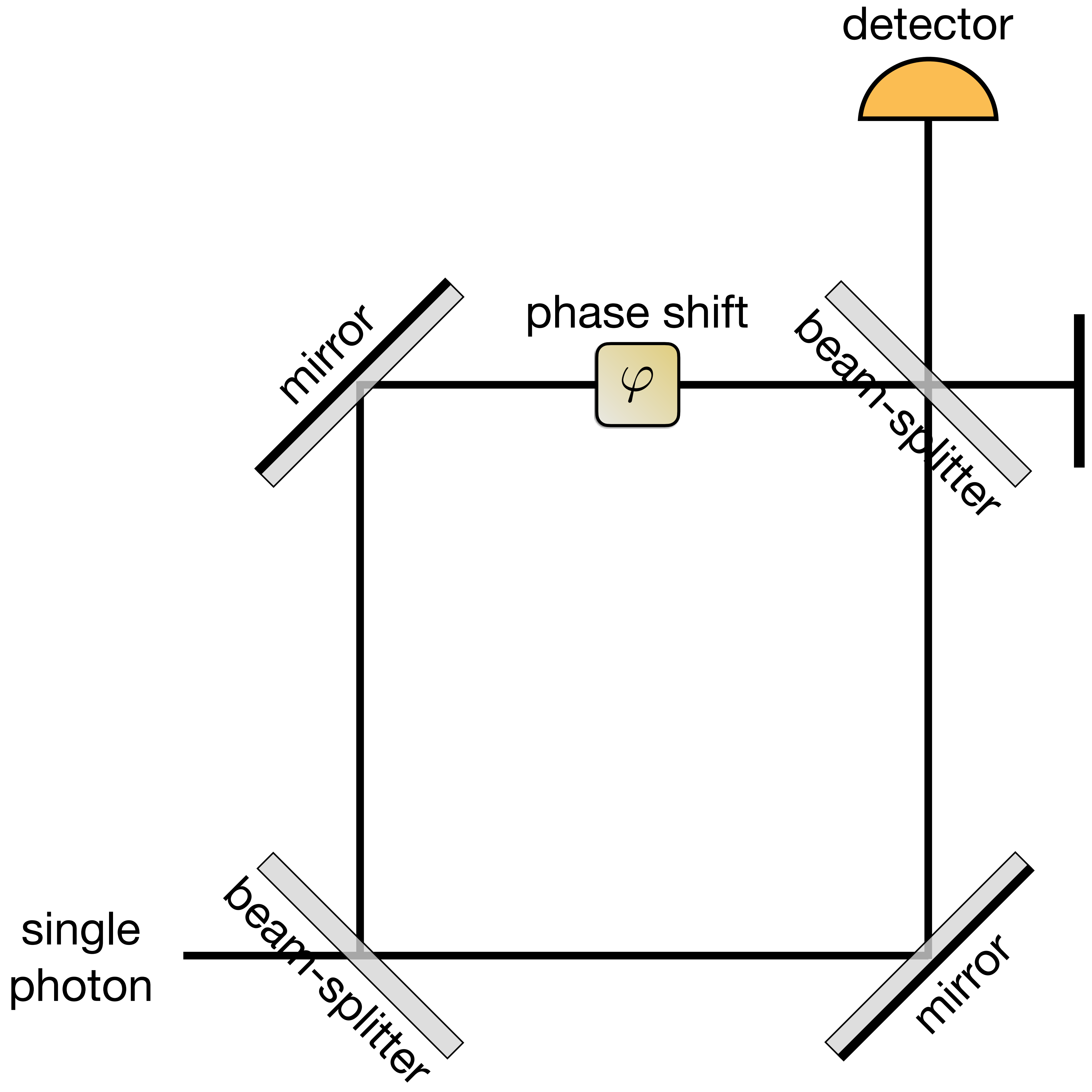
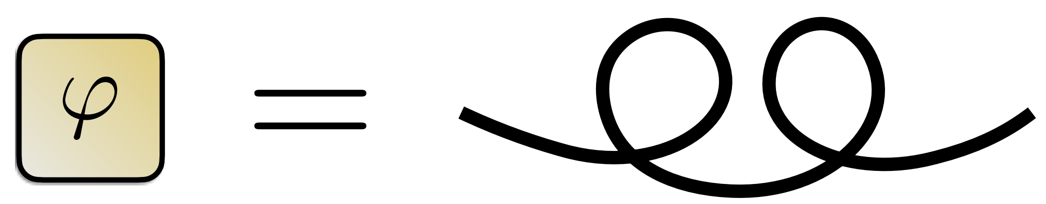
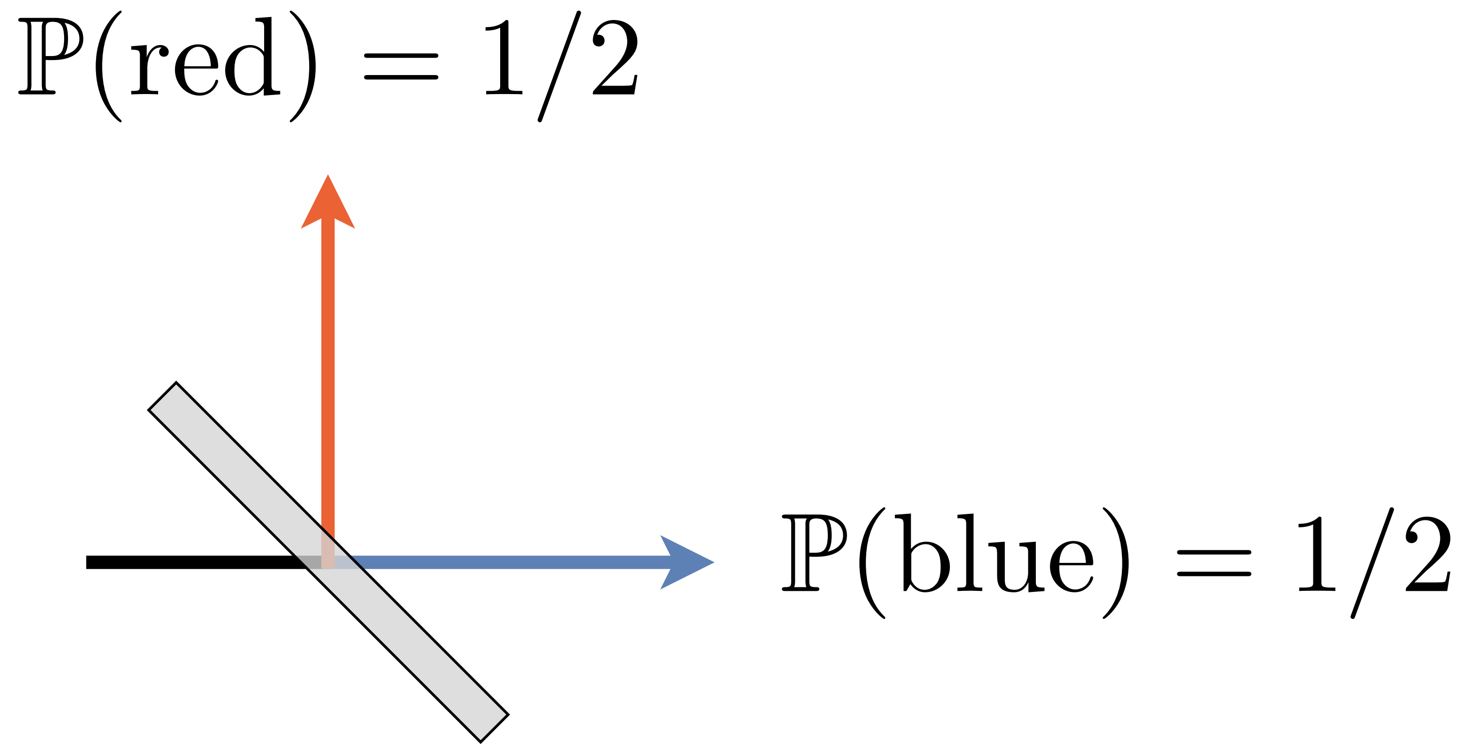
When Classical Mechanics Fails



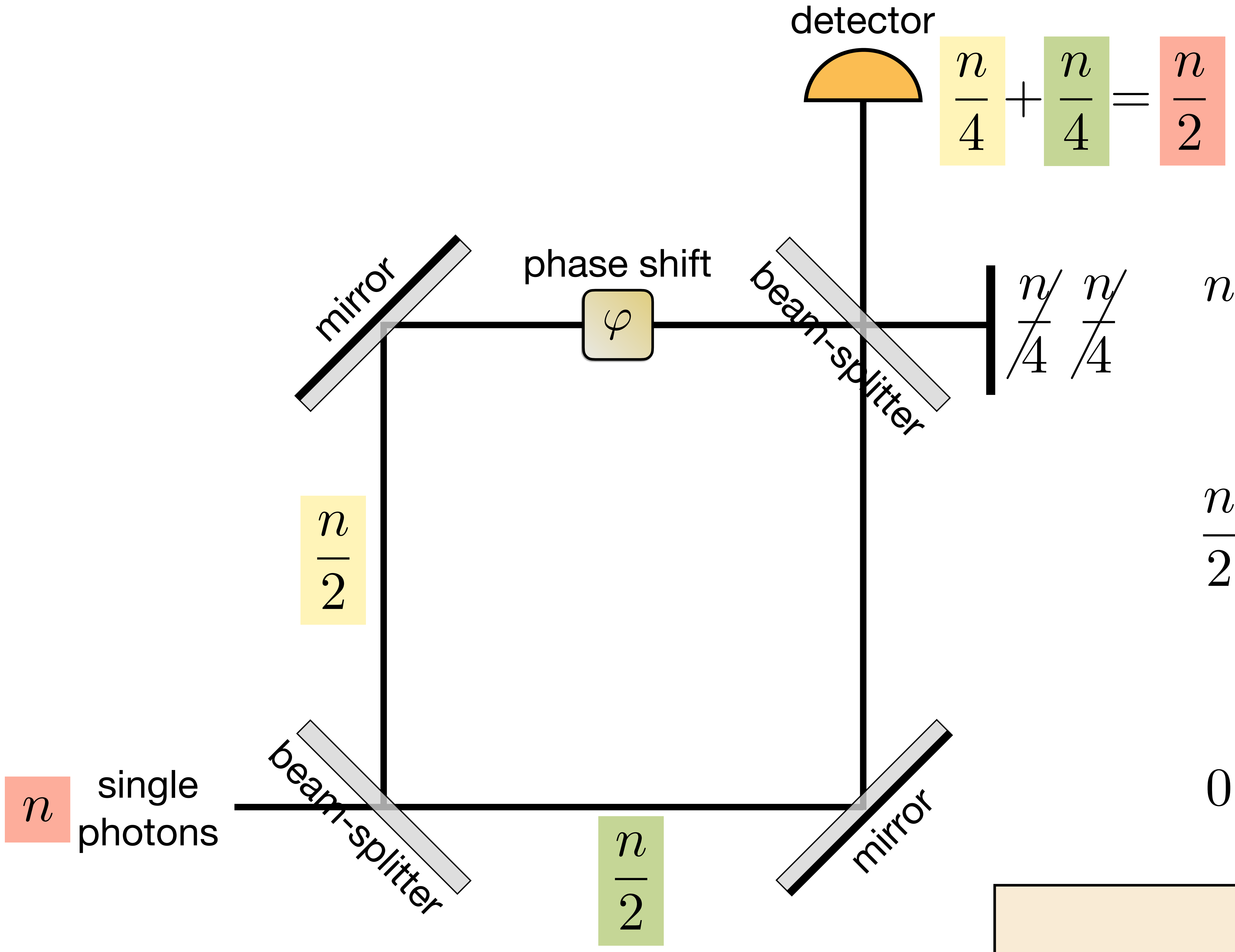
Mach-Zehnder Interferometer





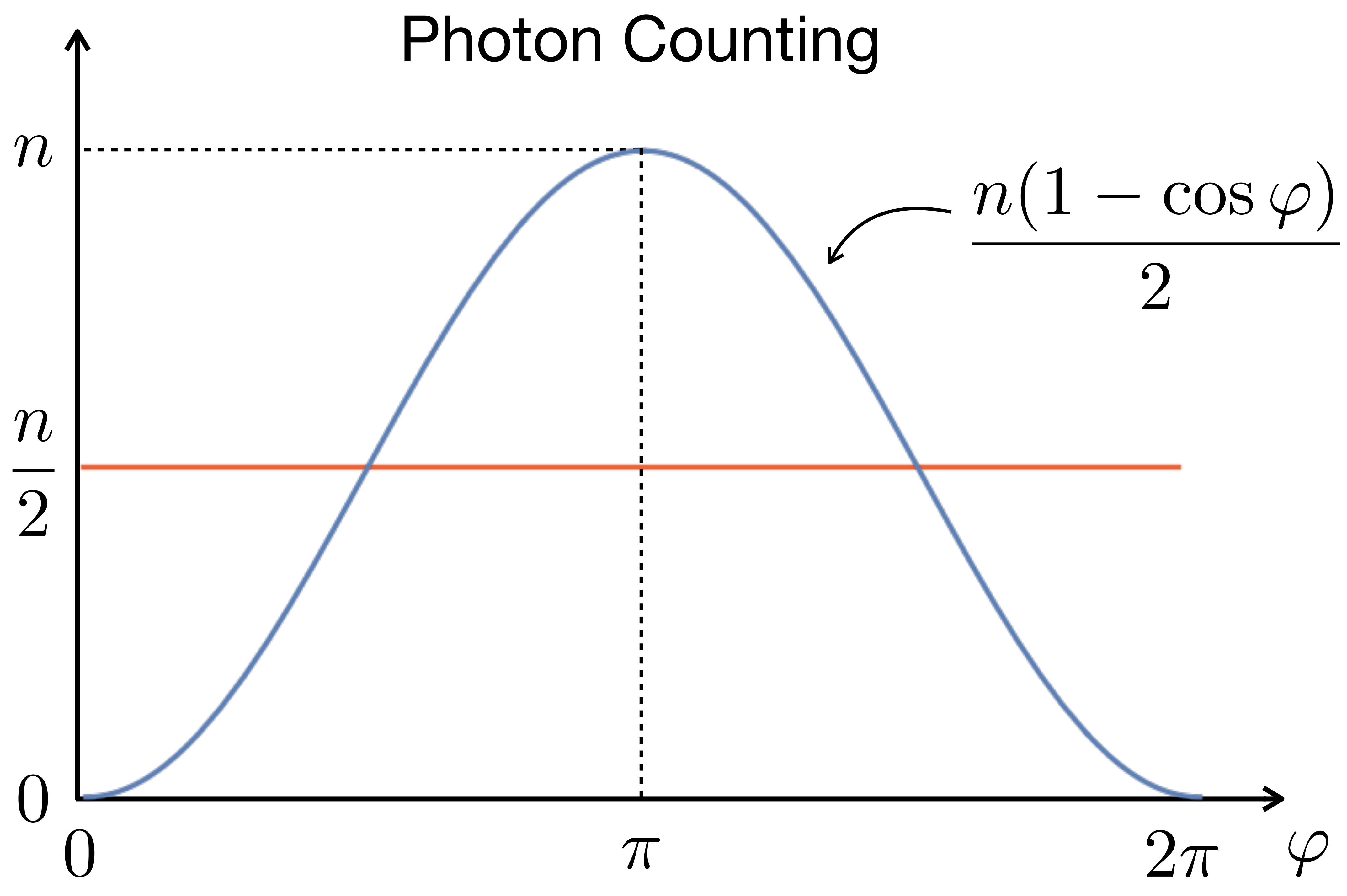
A photon takes each exit path with equal probability

When Classical Mechanics Fails



Mach-Zehnder Interferometer

The single photon behaves as if it is **interfering with itself**

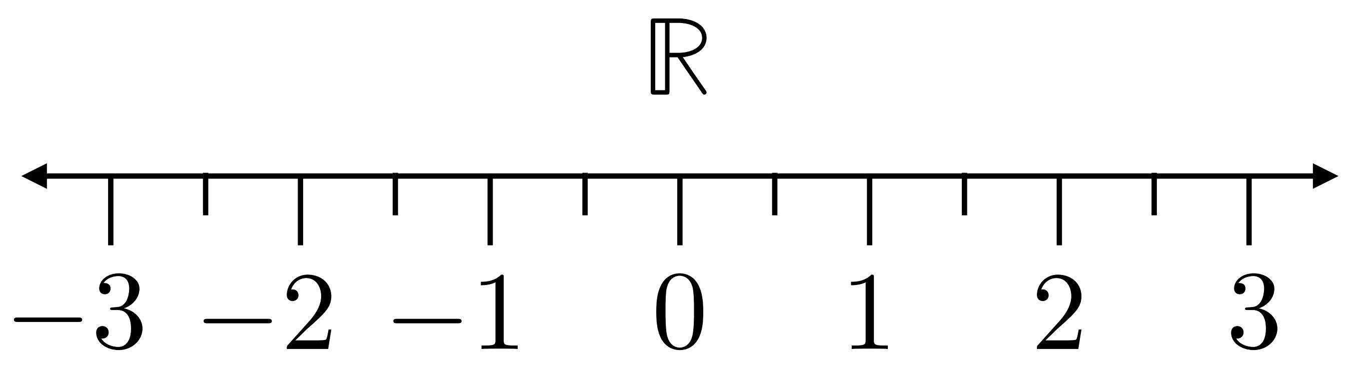


Wave-particle Duality

A particle can be thought of as both a particle and a wave

Classical Systems

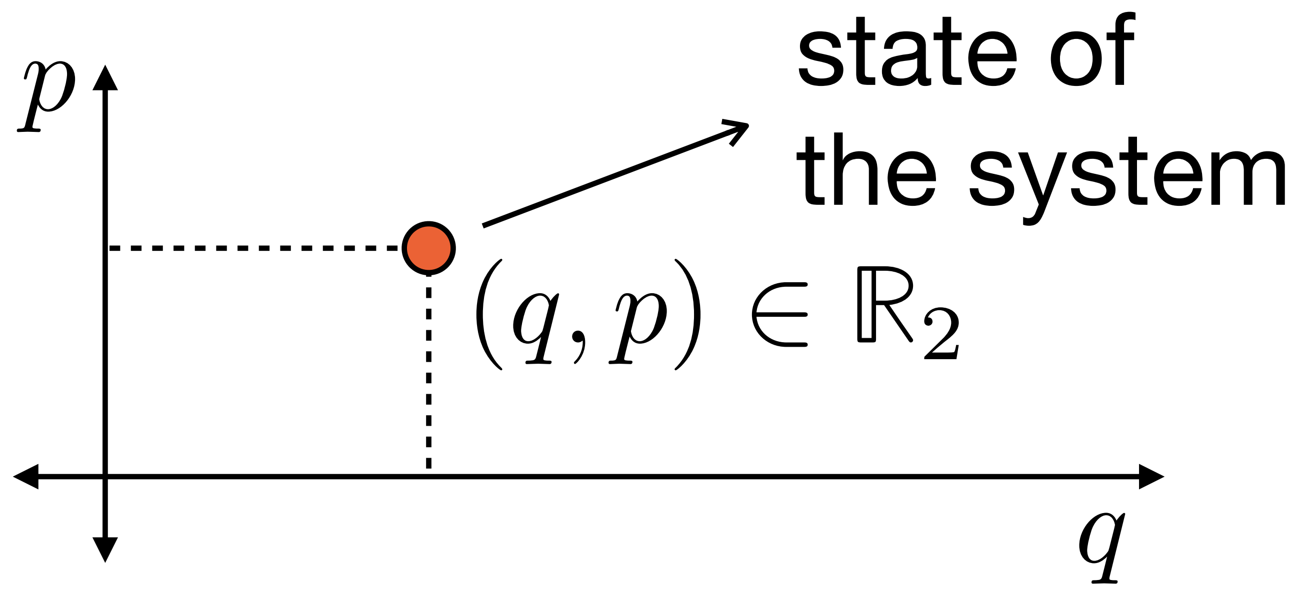
A **property** is represented by a **real-valued number**



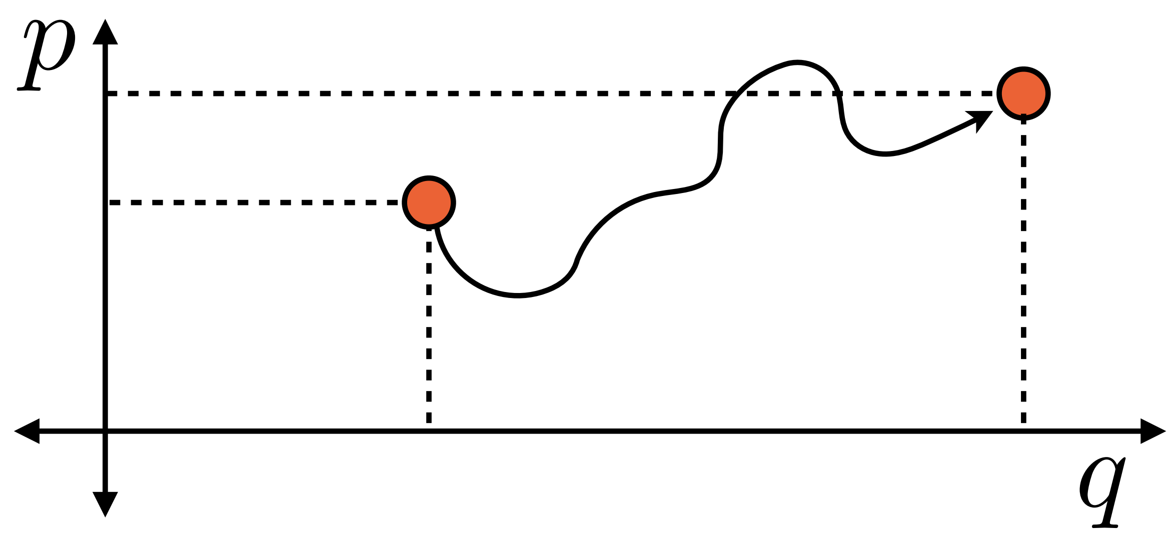
position q

momentum p

A **system** is represented by a **point in the phase space**



Evolution of a property/state is **deterministic**



$$\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{p}{m}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = f$$

Dirac Notation

- A **column vector** is represented with a "**ket**", e.g., $|x\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$
- A **row vector** is represented with a "**bra**", e.g., $\langle y| = \begin{bmatrix} c & d \end{bmatrix}$
- A **ket** can be **transformed** into a **bra** as follows: $|x\rangle \rightarrow \langle x| = \begin{bmatrix} a^* & b^* \end{bmatrix} = |x\rangle^\dagger$ (conjugate transpose)
- The **inner product** is represented as a **bra-ket** $\langle y|x\rangle = \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = a \cdot c + b \cdot d$
- The **outer product** as $|x\rangle\langle y| = \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} c & d \end{bmatrix} = \begin{bmatrix} ac & ad \\ bc & bd \end{bmatrix}$ E.g., $|x\rangle\langle y| \cdot |z\rangle = |x\rangle \langle y|z\rangle = \langle y|z\rangle \, |x\rangle$
- We consider the **computational basis** $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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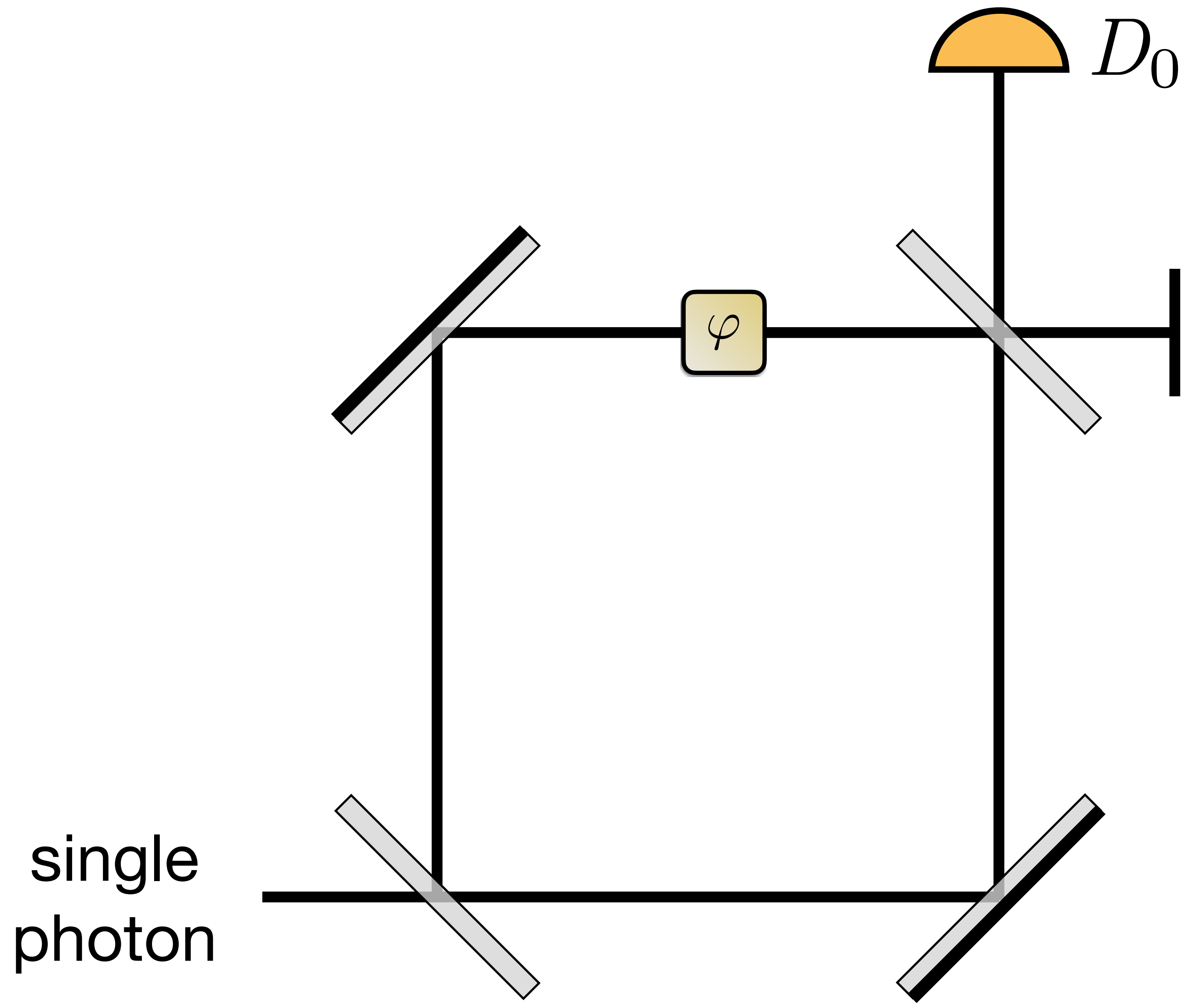
Quantum Observable

A **classical property** is represented by a **number**

A **quantum property**, known as **quantum observable**, is represented by a **matrix**

- Those matrices have **real eigenvalues** and represent the **possible outcomes of measurements**.
- E.g., **Photon detection** is a binary property that can be represented by a matrix:



$$D_0 = |0\rangle\langle 0| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$ Eigenvalues: 0 and 1

Detection: 1 Non-detection: 0

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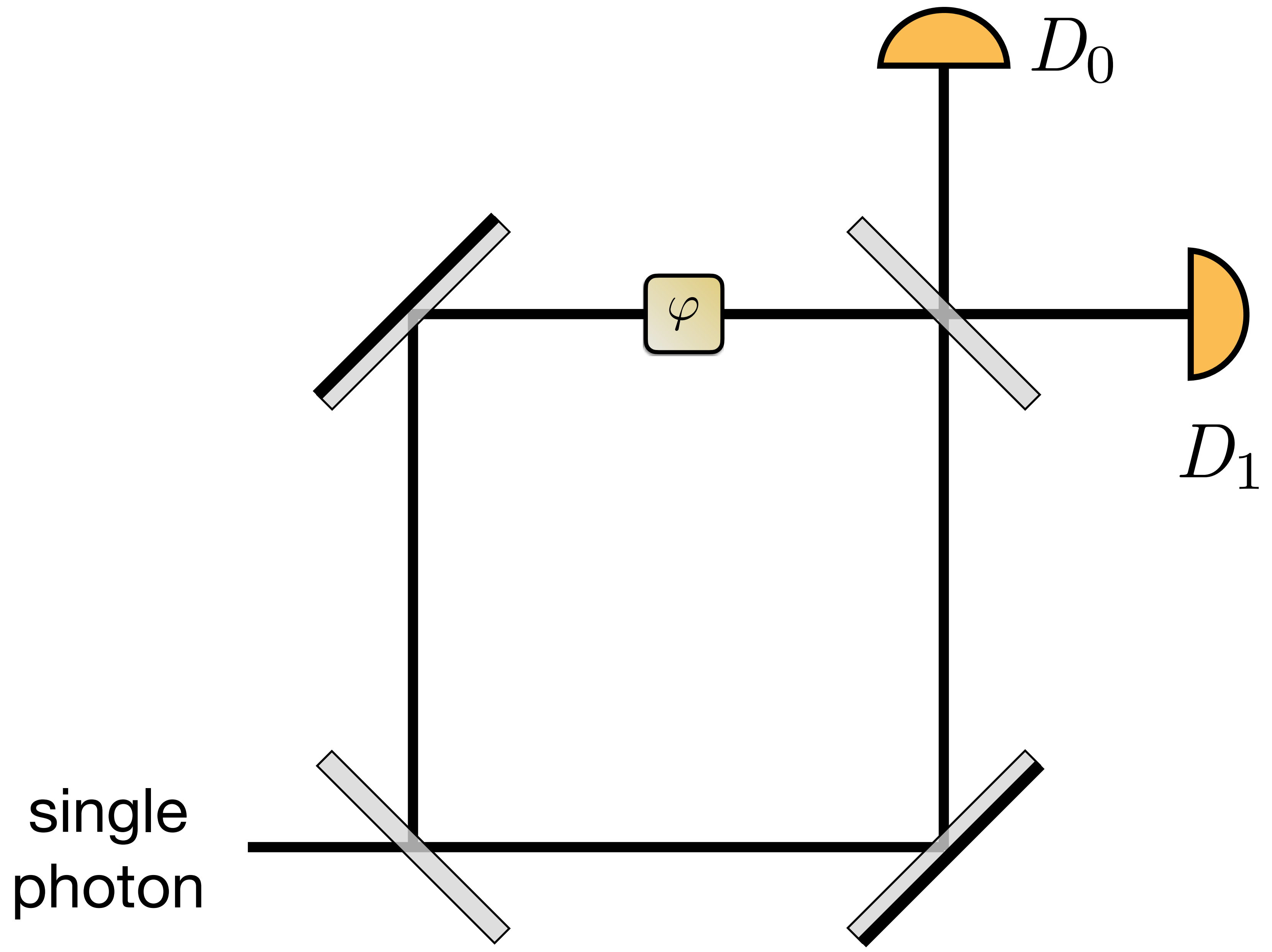

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Detection: 1 Non-detection: 0

$$D_1 = |1\rangle\langle 1| = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$ Eigenvalues: 0 and 1

$$D = \lambda_0 D_0 + \lambda_1 D_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \lambda_0 \neq \lambda_1 \qquad \begin{matrix} \lambda_0 = 1 \\ \lambda_1 = -1 \end{matrix}$$

eigenvalues quantum measurements

Quantum Observable

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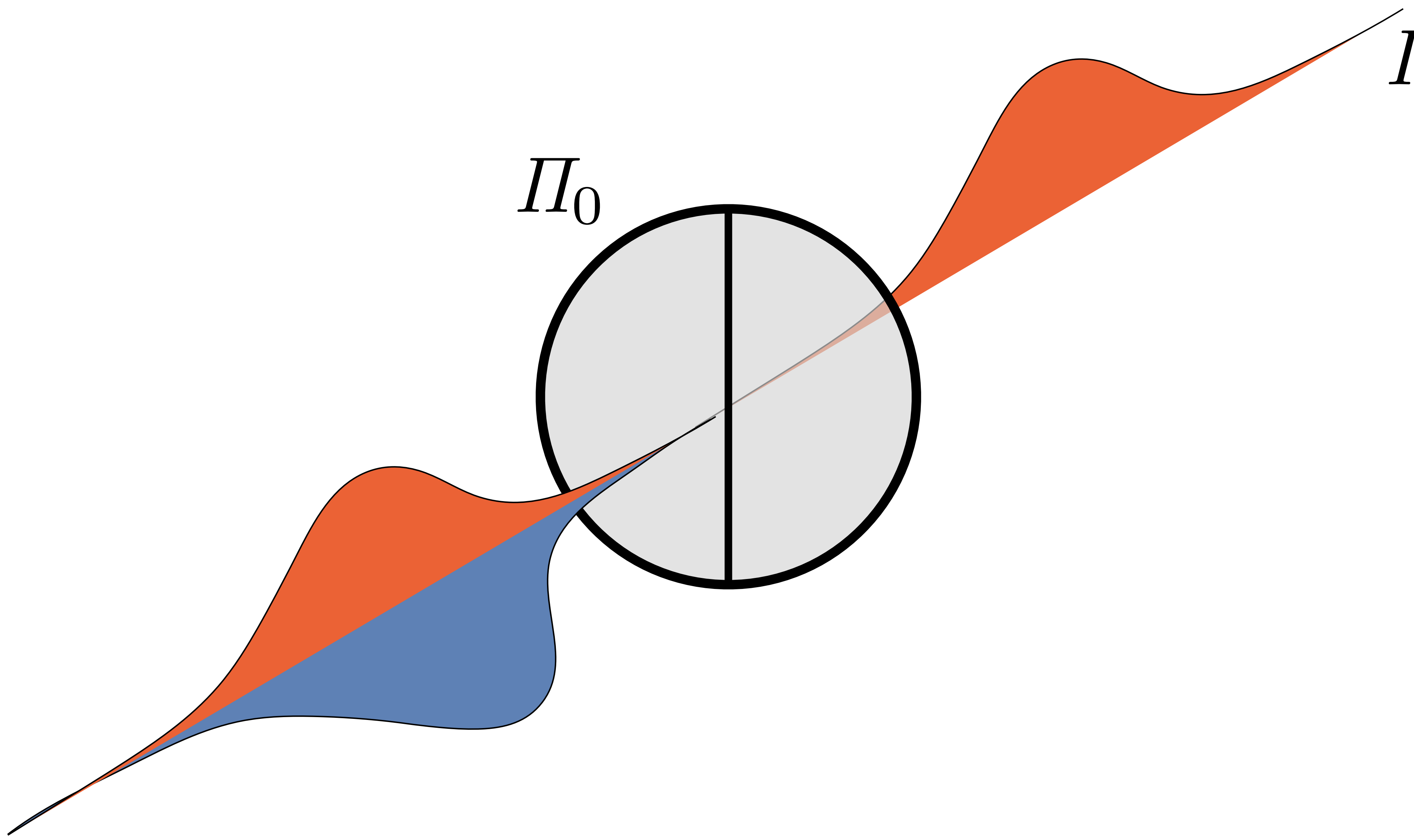

$$\Pi_0 = |0\rangle\langle 0| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$ Eigenvalues: 0 and 1

Pass: 1 No pass: 0

Quantum Observable

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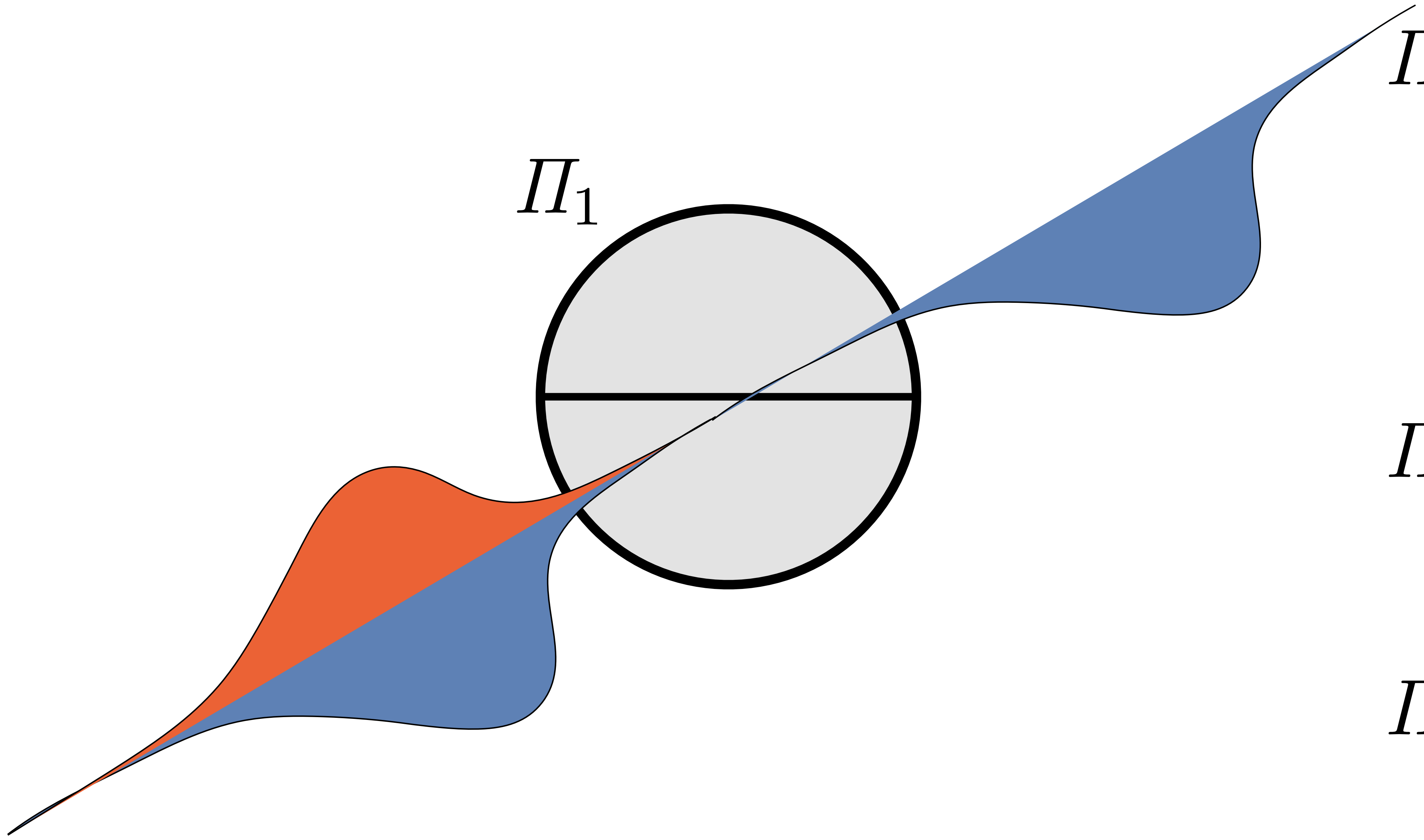

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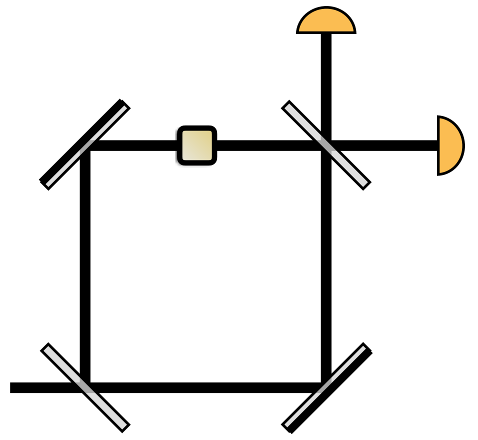
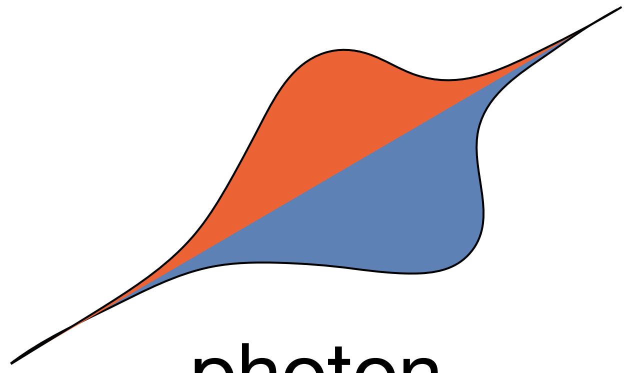
$$\Pi = \lambda_0 \Pi_0 + \lambda_1 \Pi_1 = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \qquad \lambda_0 \neq \lambda_1 \qquad \begin{matrix}\lambda_0 = 1\\ \lambda_1 = -1\end{matrix}$$

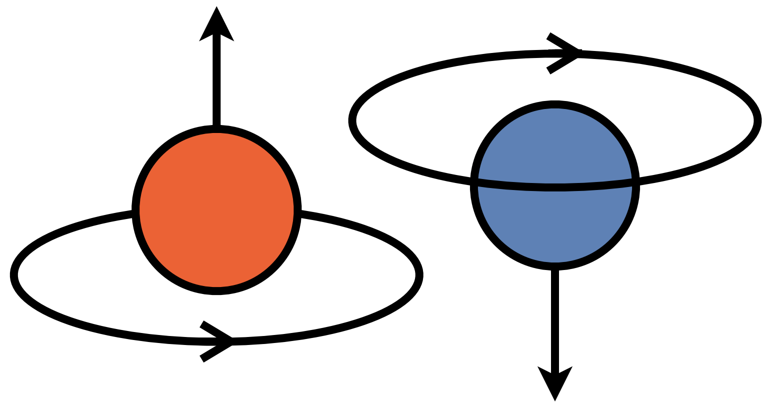
eigenvalues (λ_0, λ_1); quantum measurements (Π_0, Π_1)

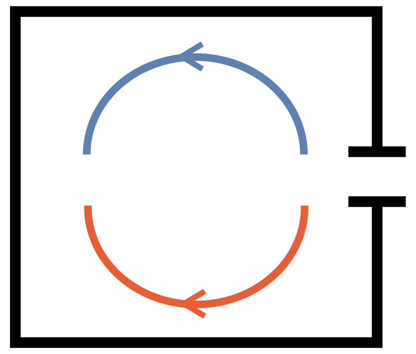
Different Types of Observables

Observable with **two** possible values
$$\begin{bmatrix} \star & \star \\ \star & \star \end{bmatrix}$$

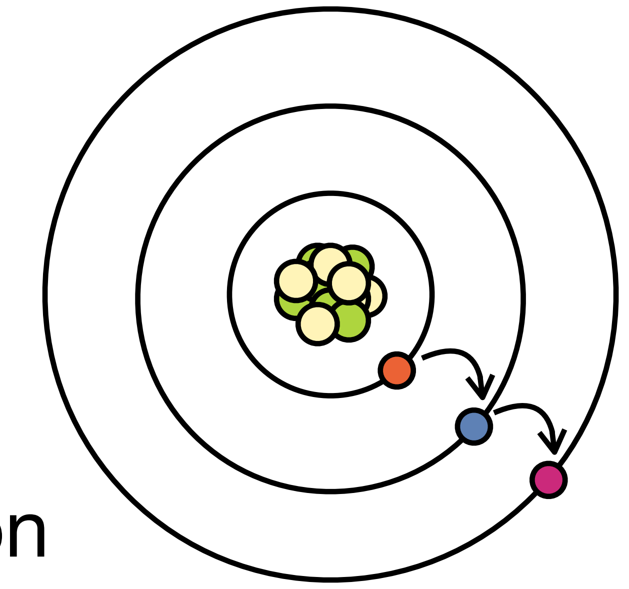


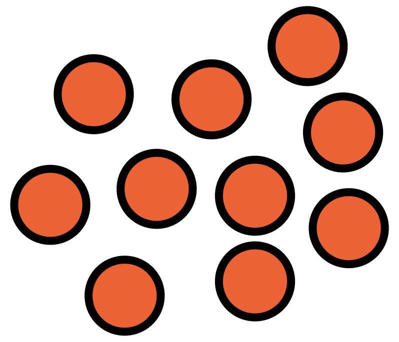




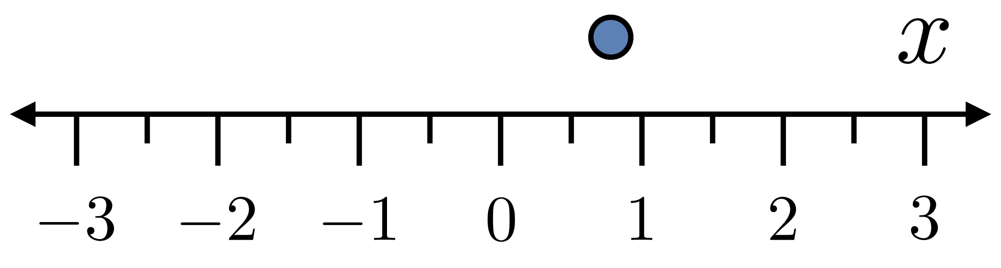


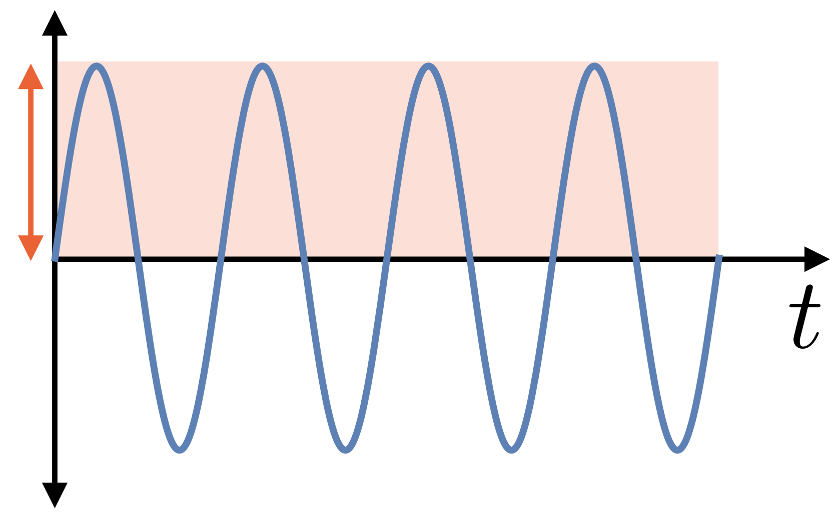
Observable with **finite** possible values
$$\begin{bmatrix} \star & \cdots & \star \\ \vdots & \ddots & \vdots \\ \star & \cdots & \star \end{bmatrix}$$





Observable with **infinite** possible values
$$\begin{bmatrix} \star & \cdots & \star & \cdots \\ \vdots & \ddots & \vdots & \cdots \\ \star & \cdots & \star & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$





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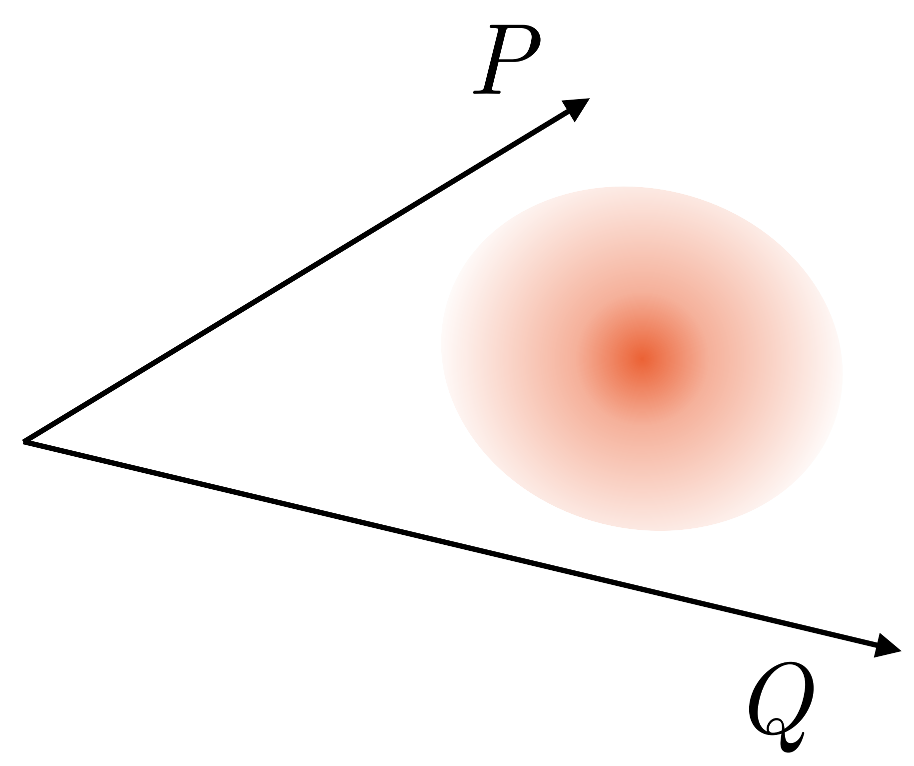
Quantum System

A **classical system** is represented by a **point** in the phase space

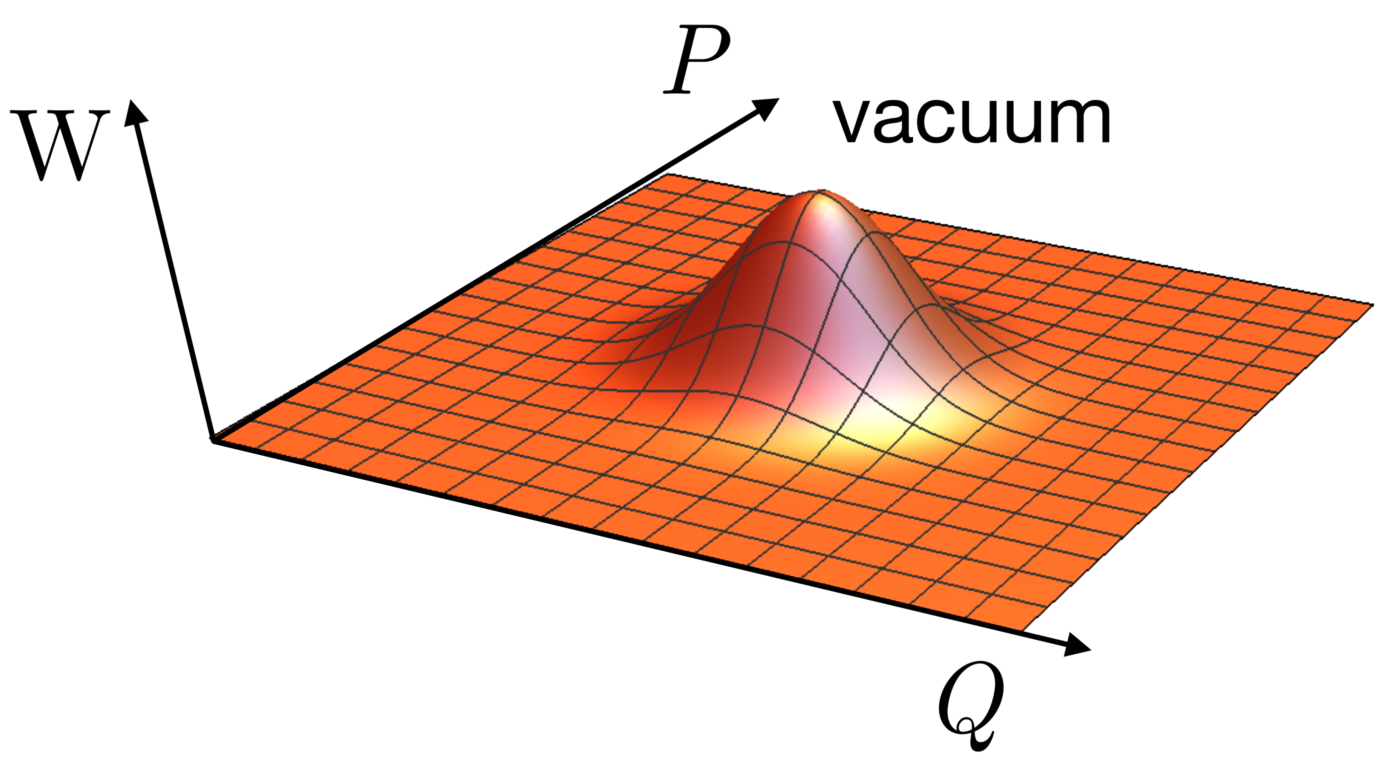


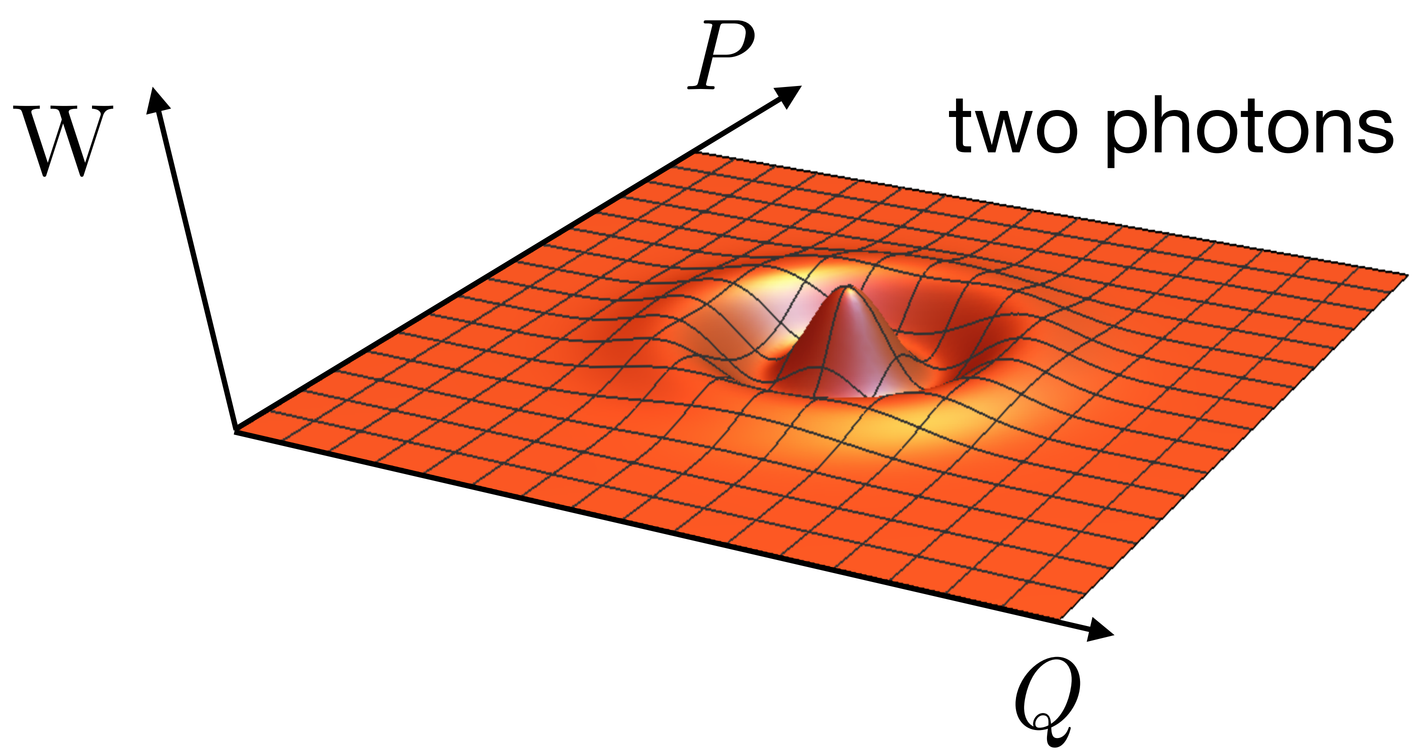
A quantum system cannot be represented as a point due to the **uncertainty principle** $\mathbb{V}(Q)\mathbb{V}(P) \geqslant \frac{\hbar^2}{4}$

Quantum System

A **classical system** is represented by a **point** in the phase space

A **quantum system** is represented by a **function** in the phase space, e.g., Wigner function





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$$\mathrm{W} = \frac{1}{4\pi}\int_{-\infty}^{\infty} dx \exp\left\{\frac{\imath x p}{2}\right\} \left\langle q - \frac{x}{2} \middle| \psi \right\rangle \left\langle \psi \middle| q + \frac{x}{2} \right\rangle$$

q : outcome of Q

p : outcome of P

x : auxiliary variable

$|\psi\rangle$: quantum state

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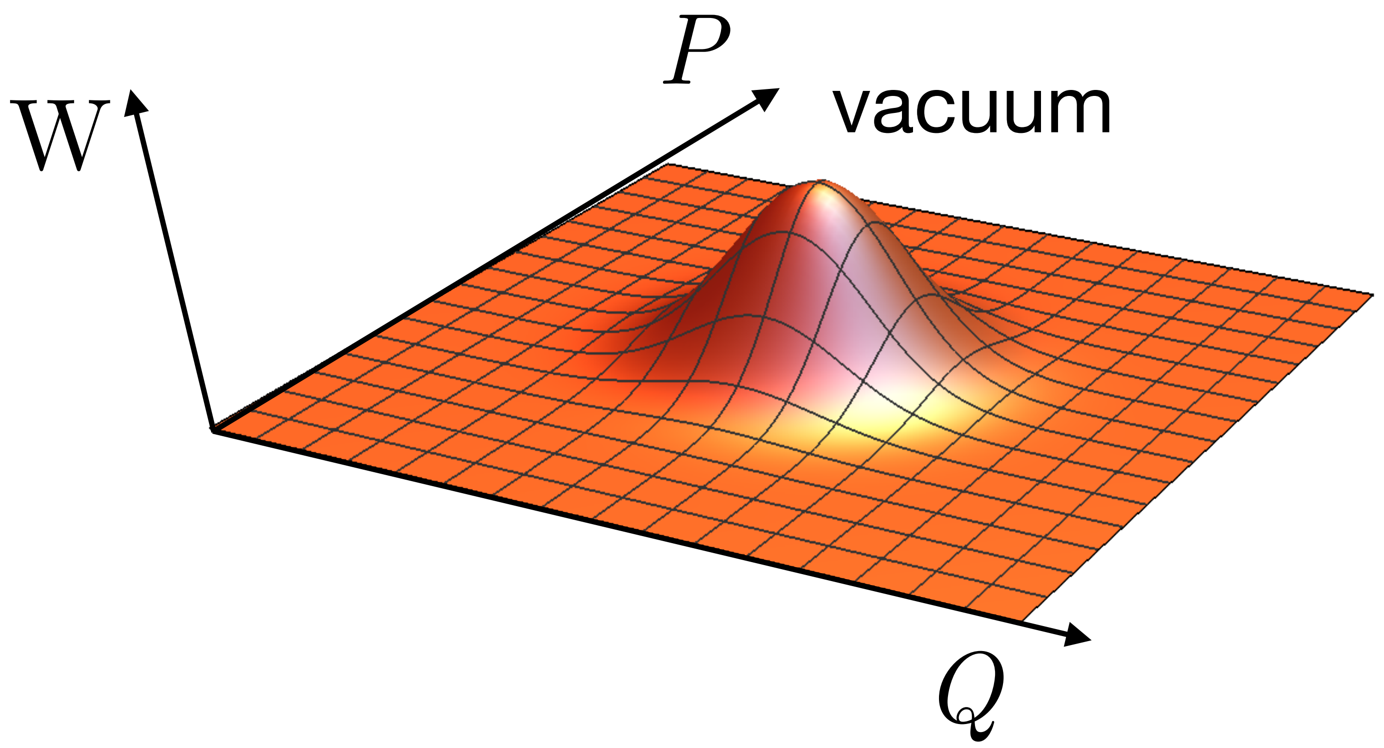

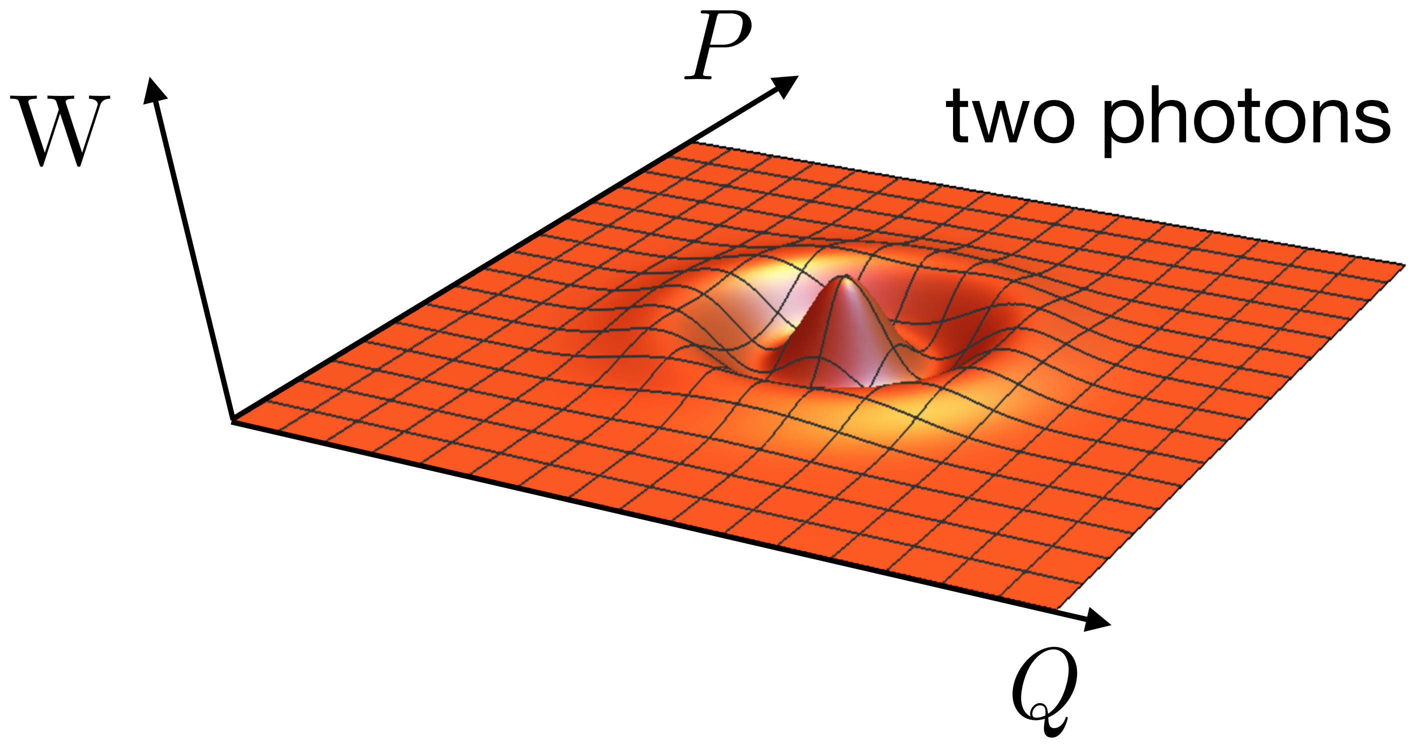

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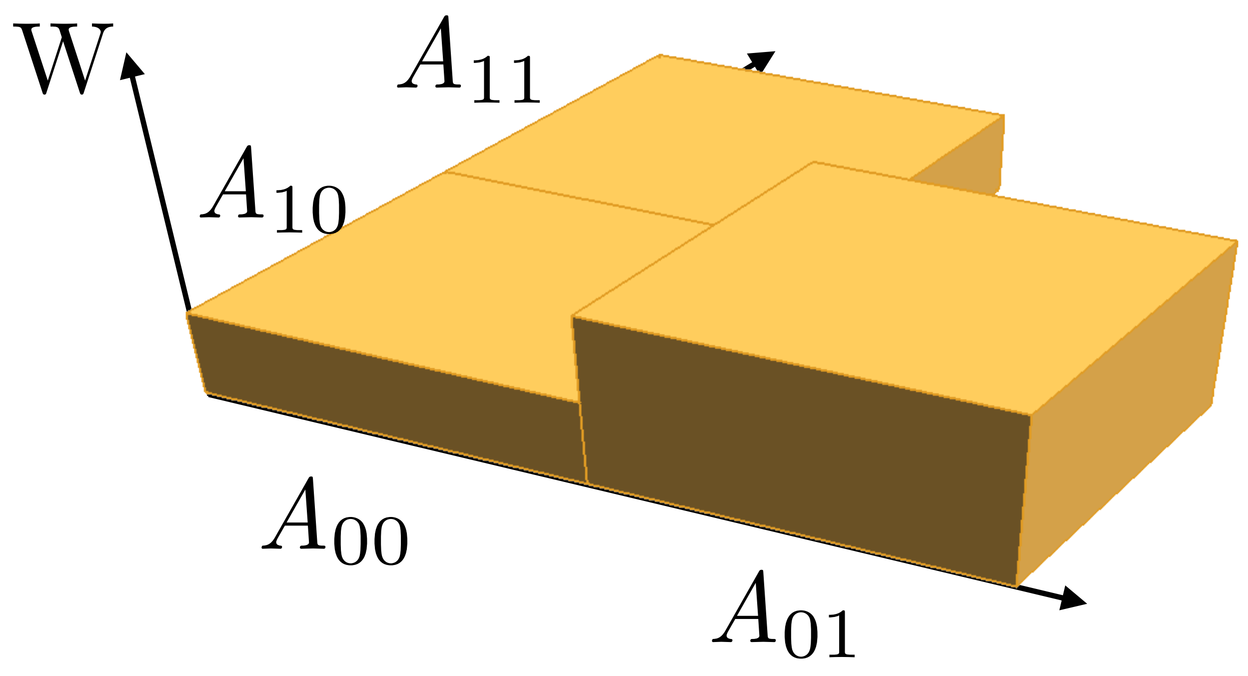


$$A_{ij} = \frac{1}{2}\left[(-1)^i Z + (-1)^j X + (-1)^{i+j} Y\right]$$

$X = |0\rangle\langle 1| + |1\rangle\langle 0|$ $\quad$ $Y = -\imath(|0\rangle\langle 1| - |1\rangle\langle 0|)$ $\quad$ $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

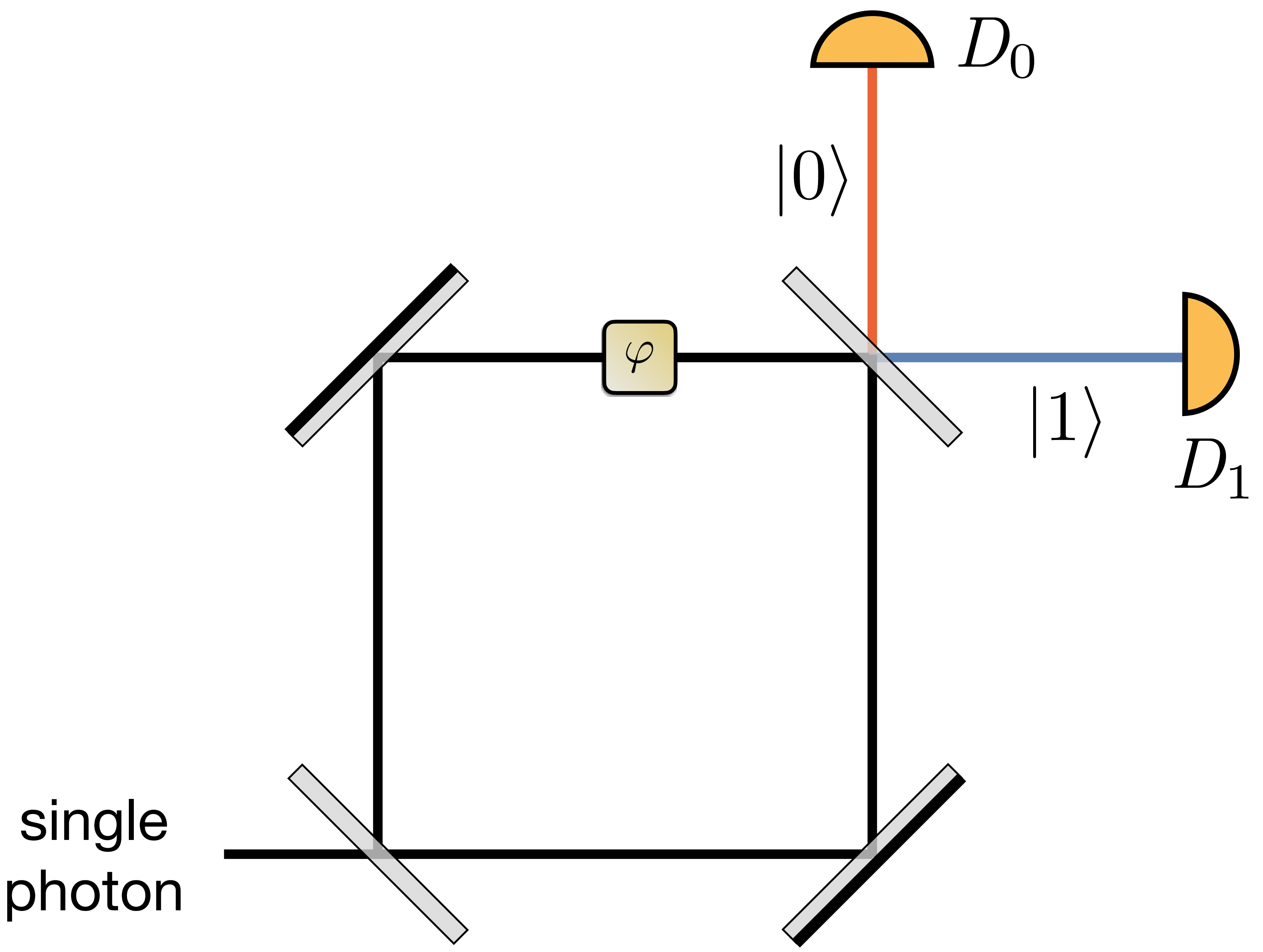
$$\mathrm{W}_{ij} = \frac{1}{2}\mathrm{tr}(|\psi\rangle\langle\psi|\, A_{ij})$$

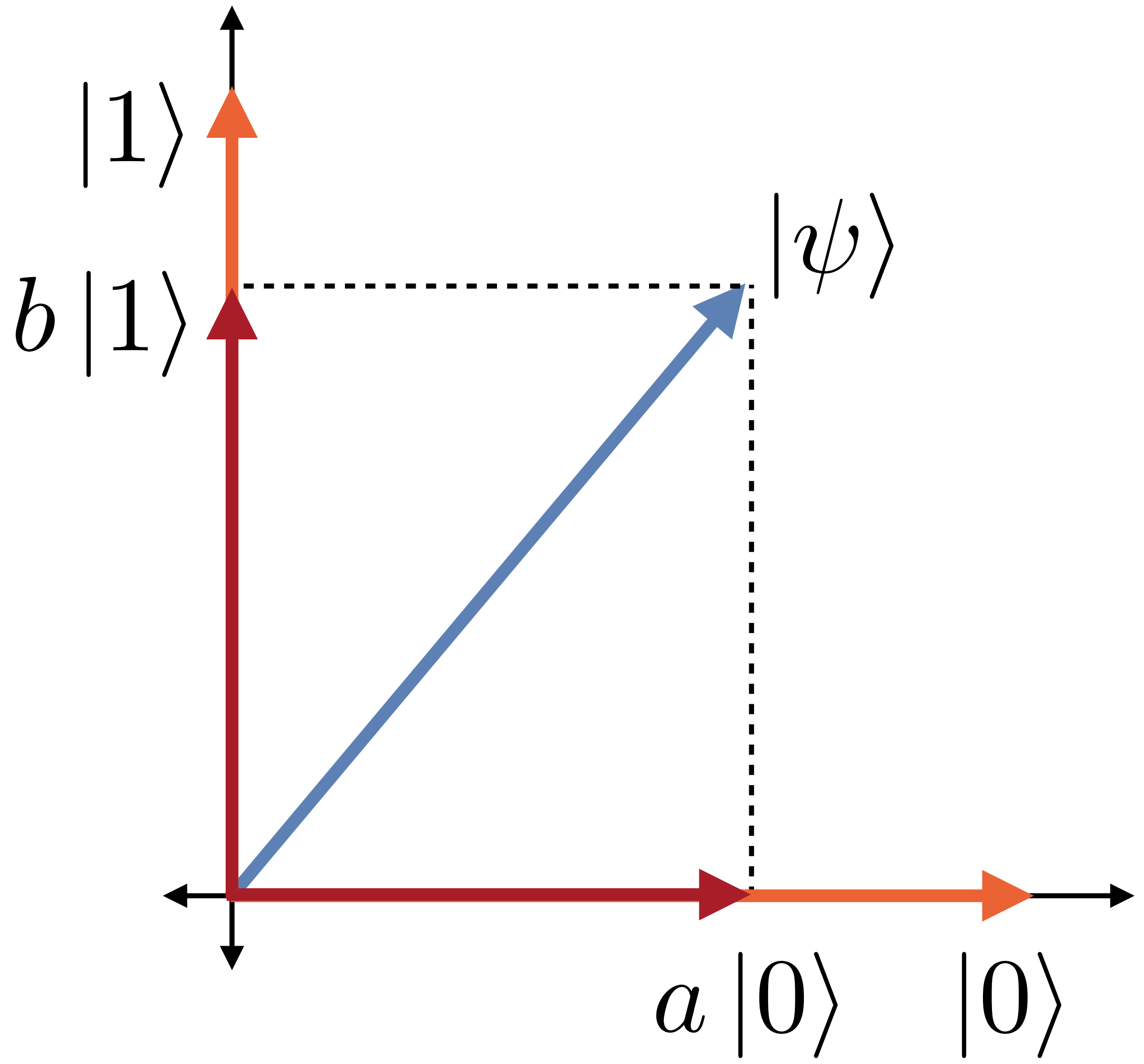
quantum state

Quantum State

A **quantum system**, known as **quantum state**, is represented by a **vector**

State Superposition: $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$

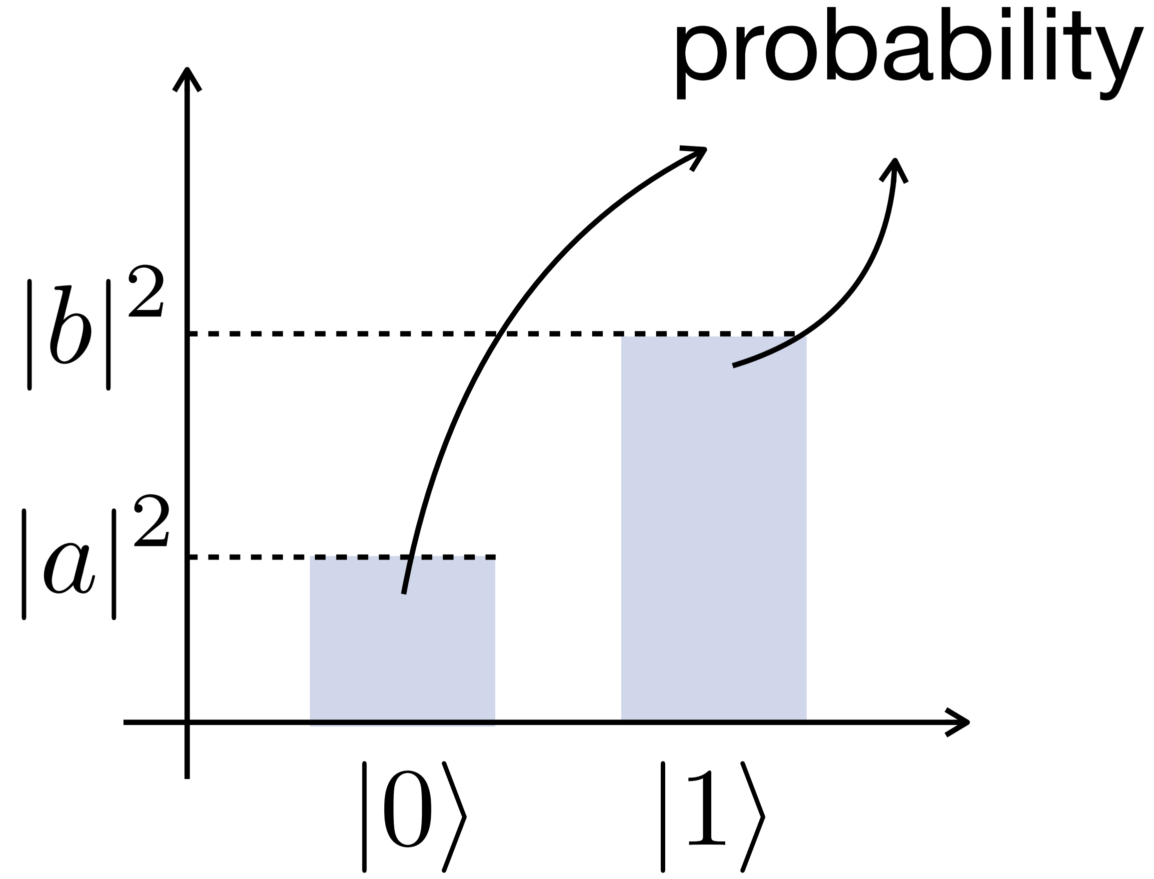




$|0\rangle \perp |1\rangle \Leftrightarrow \langle 0|1\rangle$

All states have the same length, i.e., **normalized**

$$|a|^2 + |b|^2 = 1$$



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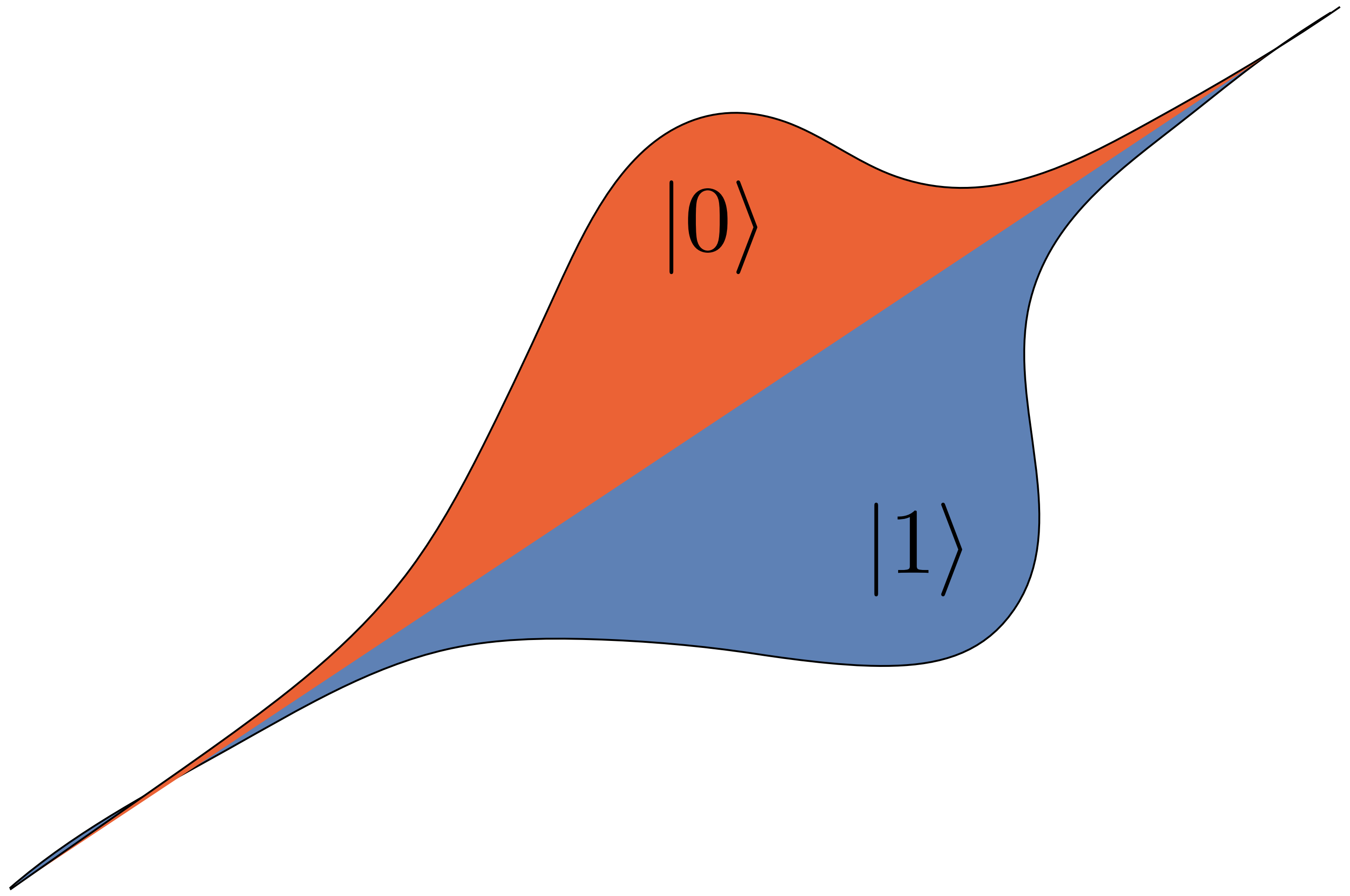

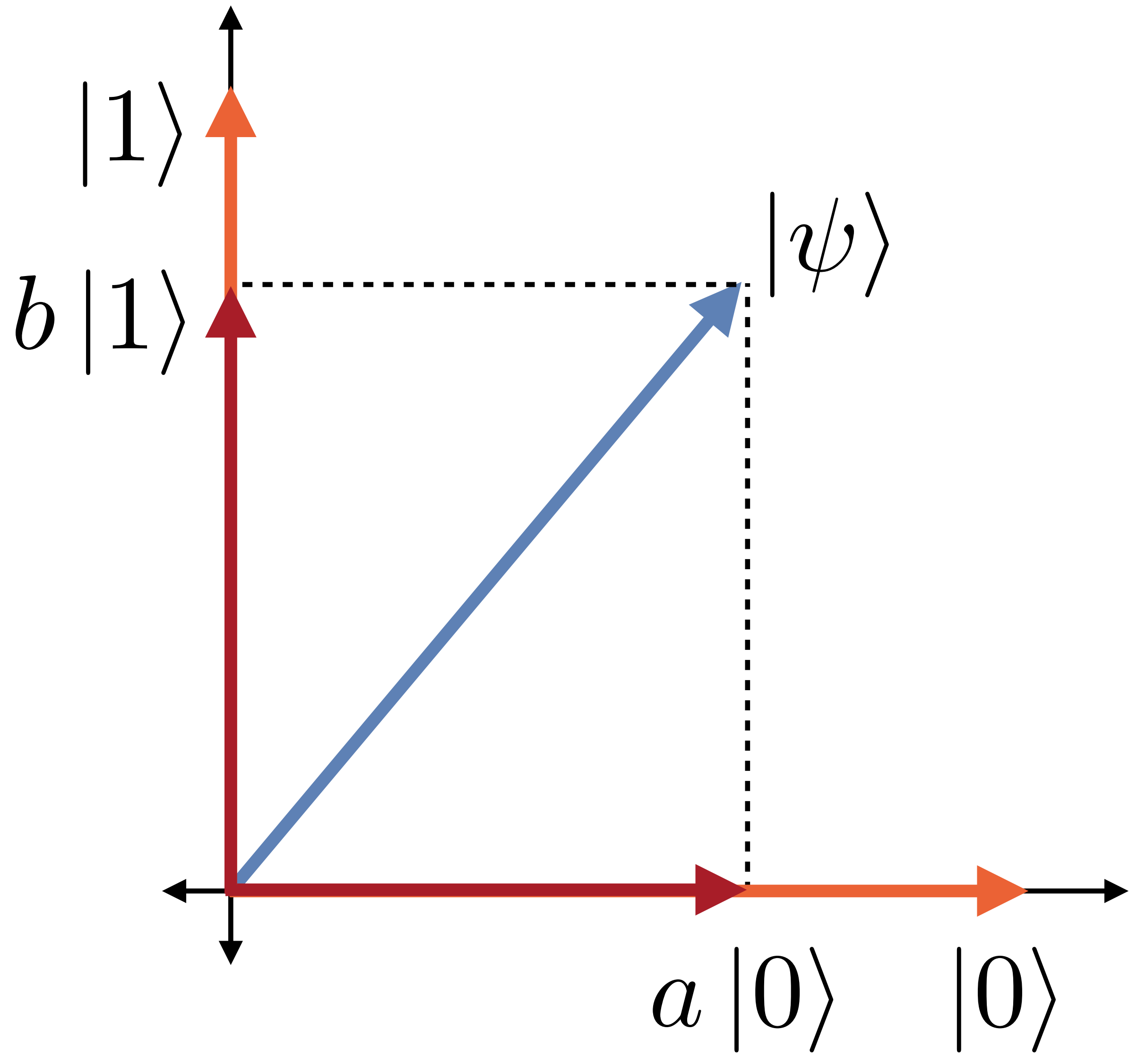

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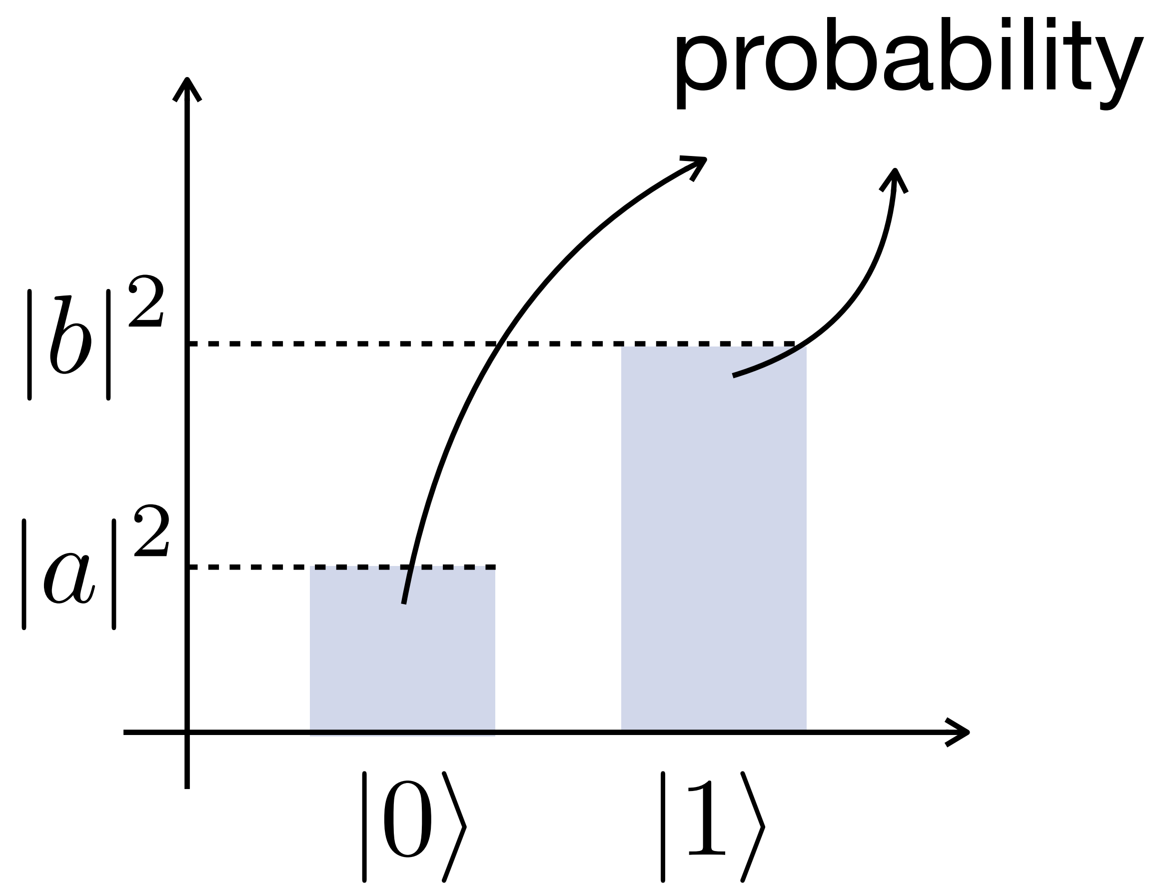
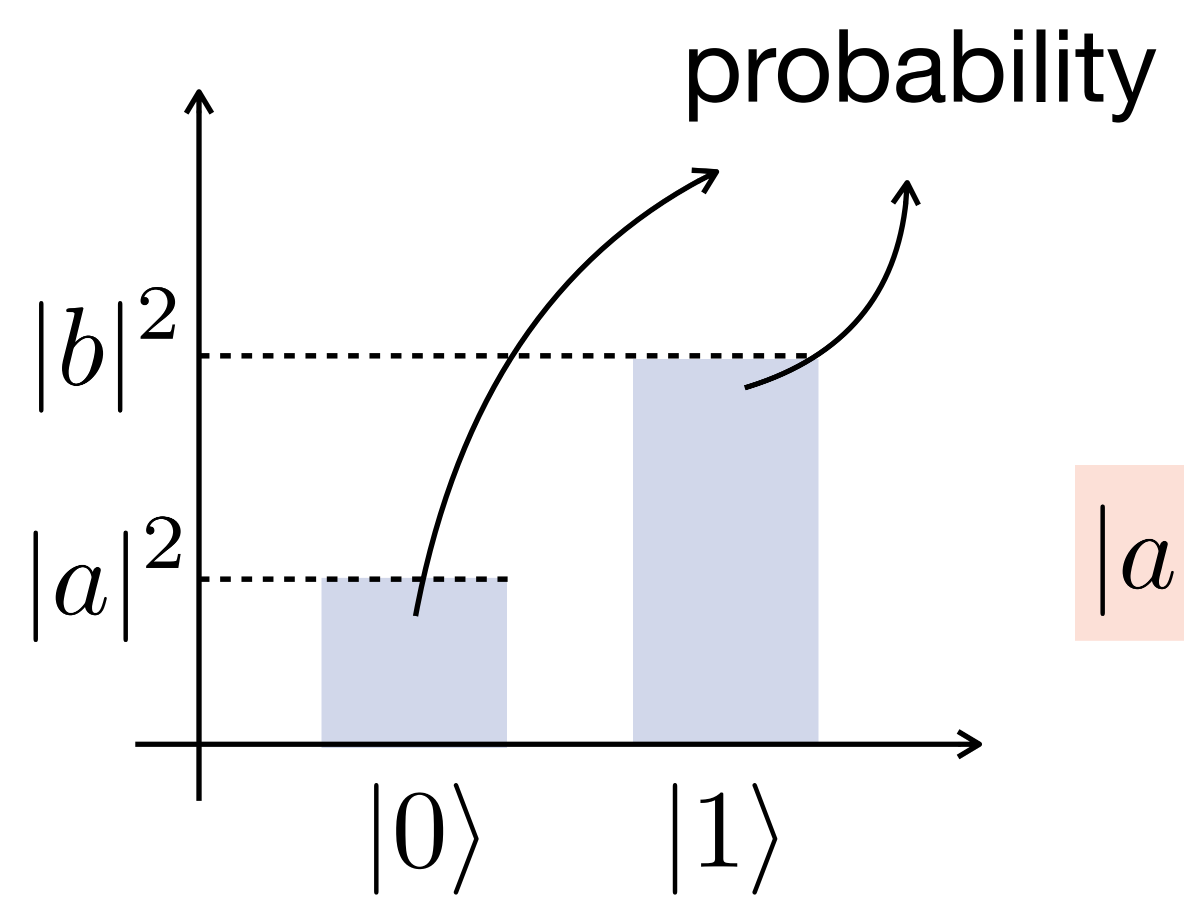


Quantum State after a Measurement

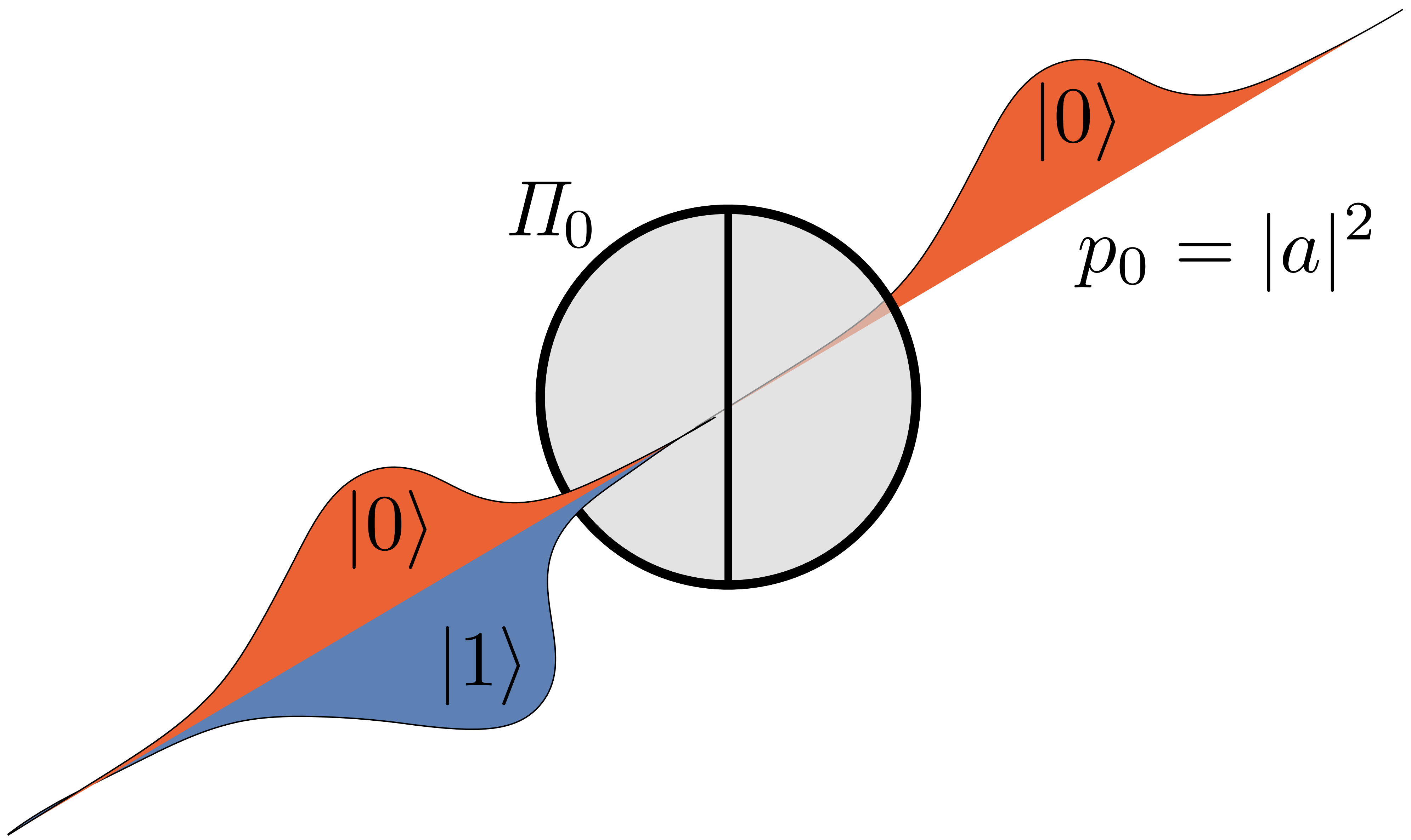
Let us have the quantum state $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$

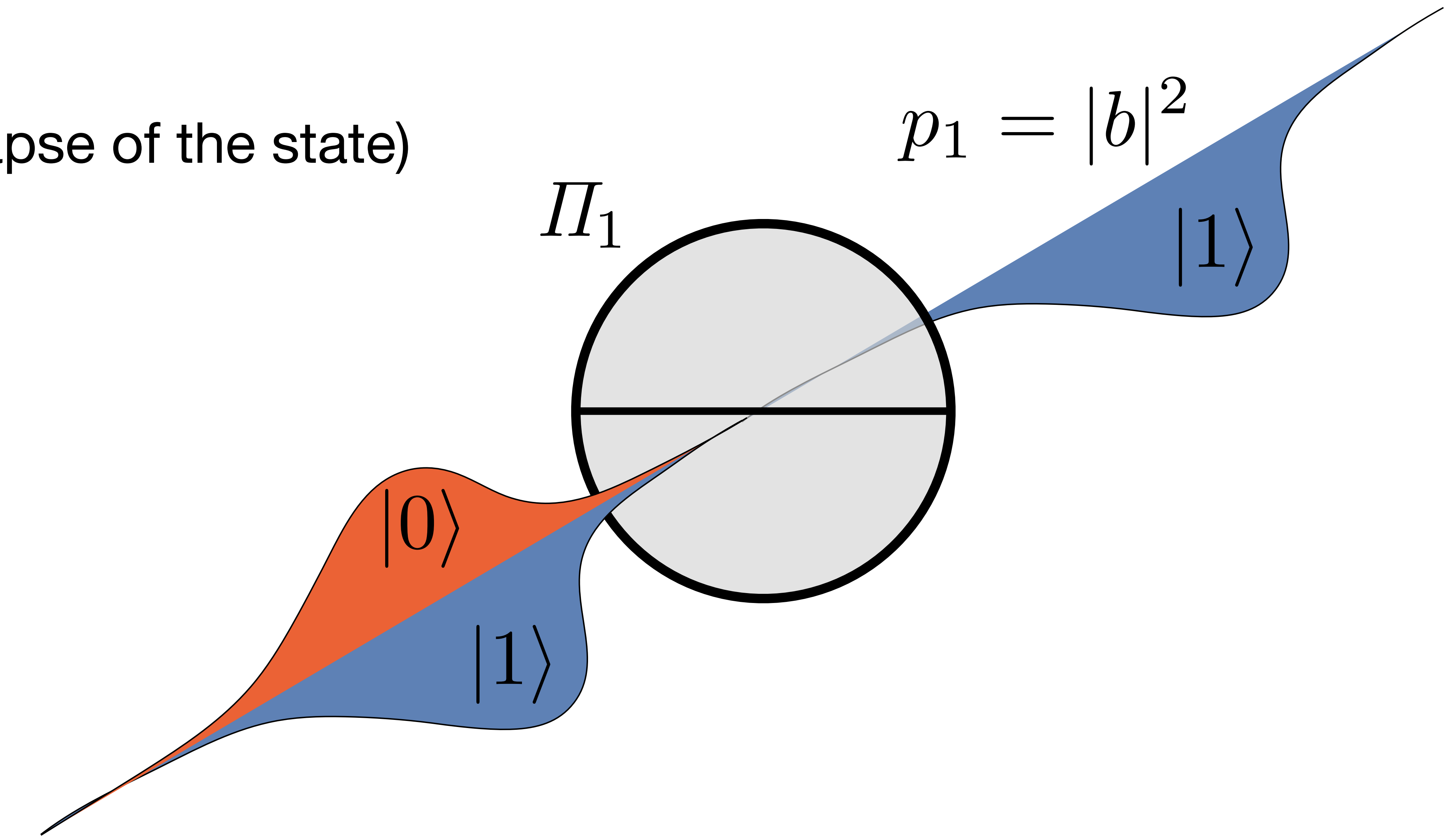


Post-measurement state $|\psi\rangle \to \dfrac{\Pi_i\,|\psi\rangle}{\sqrt{p_i}}$

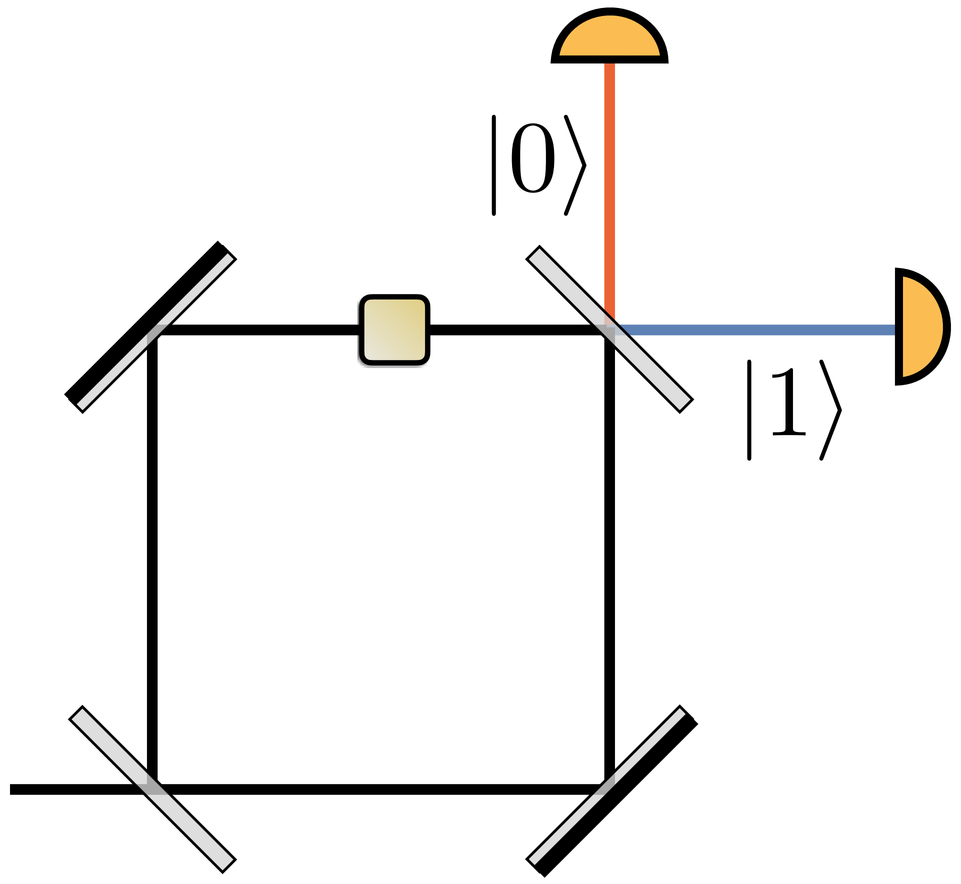
$|a|^2 + |b|^2 = 1$

(collapse of the state)



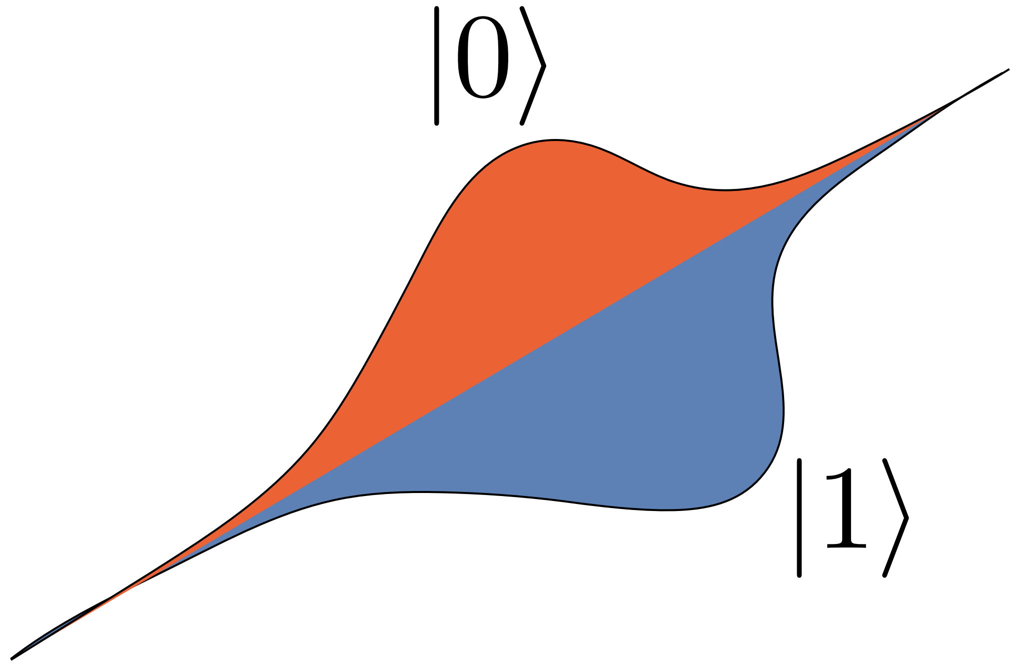


Examples of 2-dimensional States

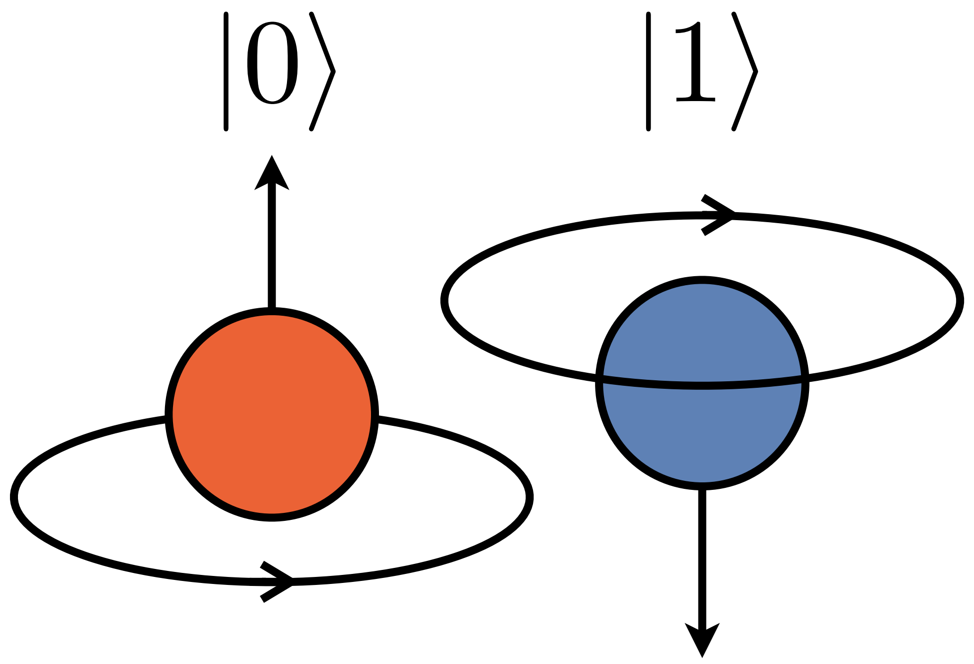


Photon Detection

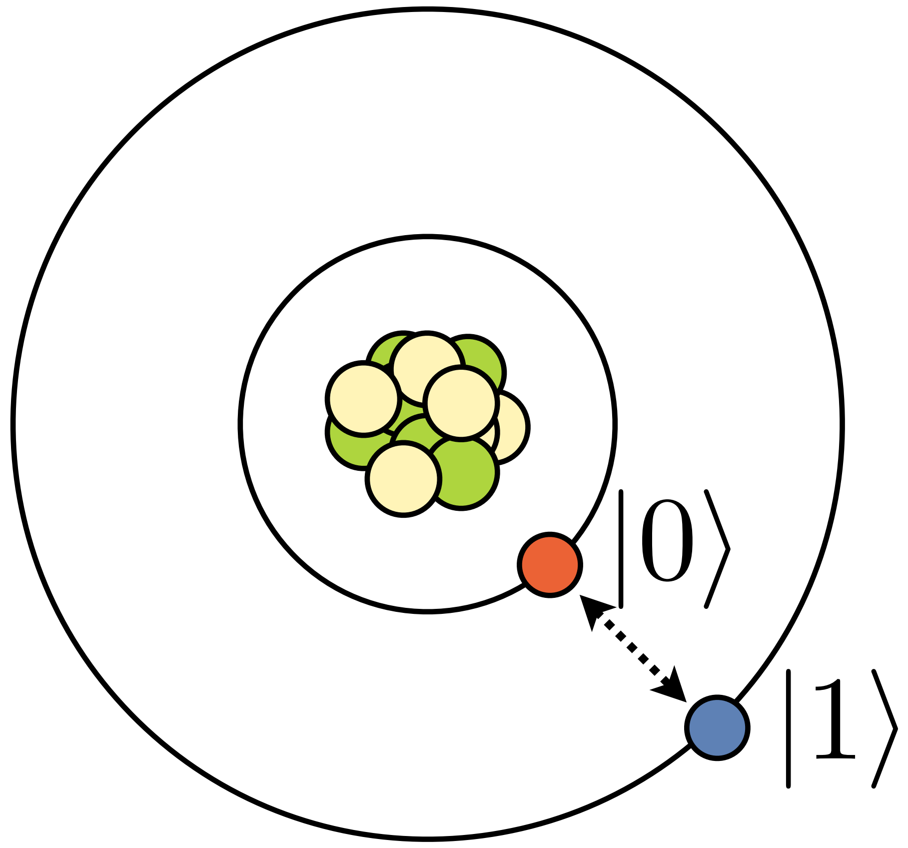
$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$



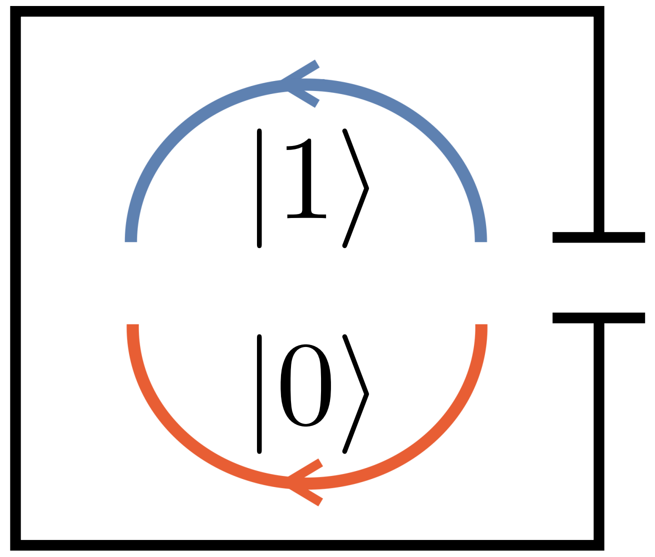
Photon Polarization



Electron Spin



Electron Excitation

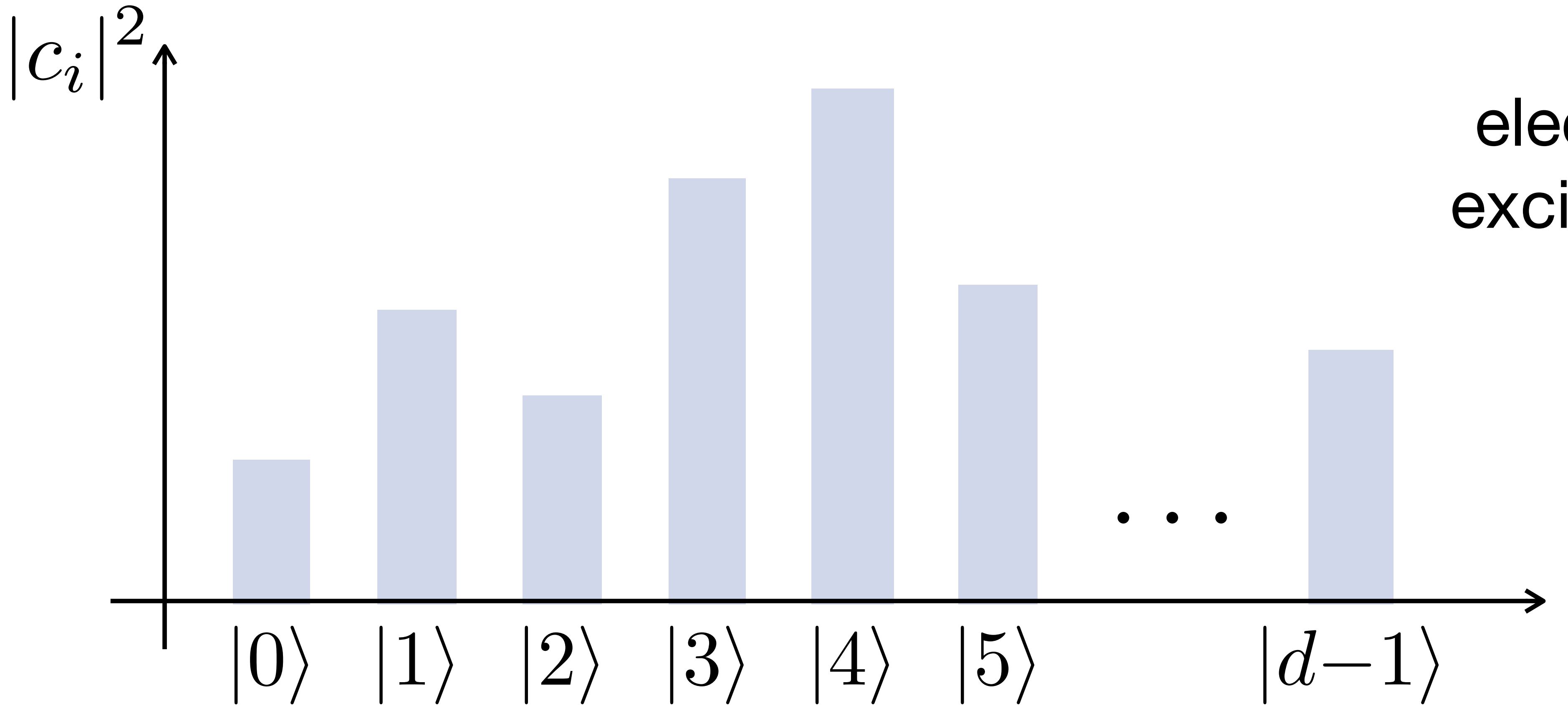


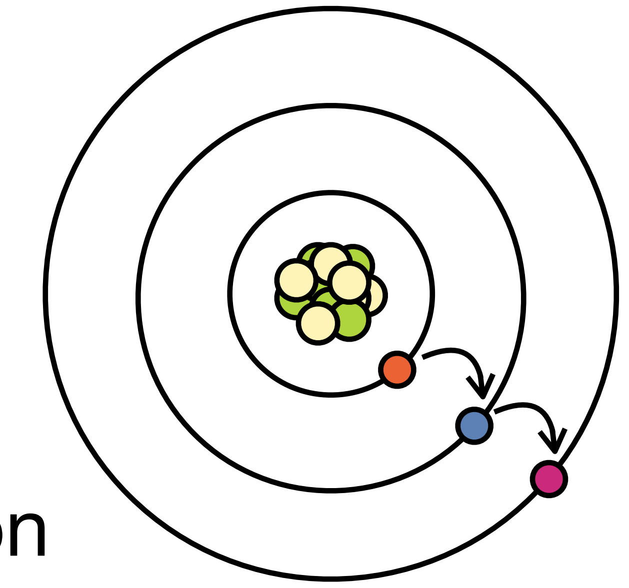
Current Flow

Higher-Dimensional States

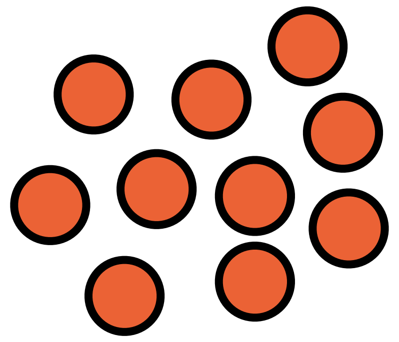
A d-dimensional quantum state

$$|\psi\rangle = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{d-1} \end{bmatrix} = \sum_{i=0}^{d-1} c_i |i\rangle$$





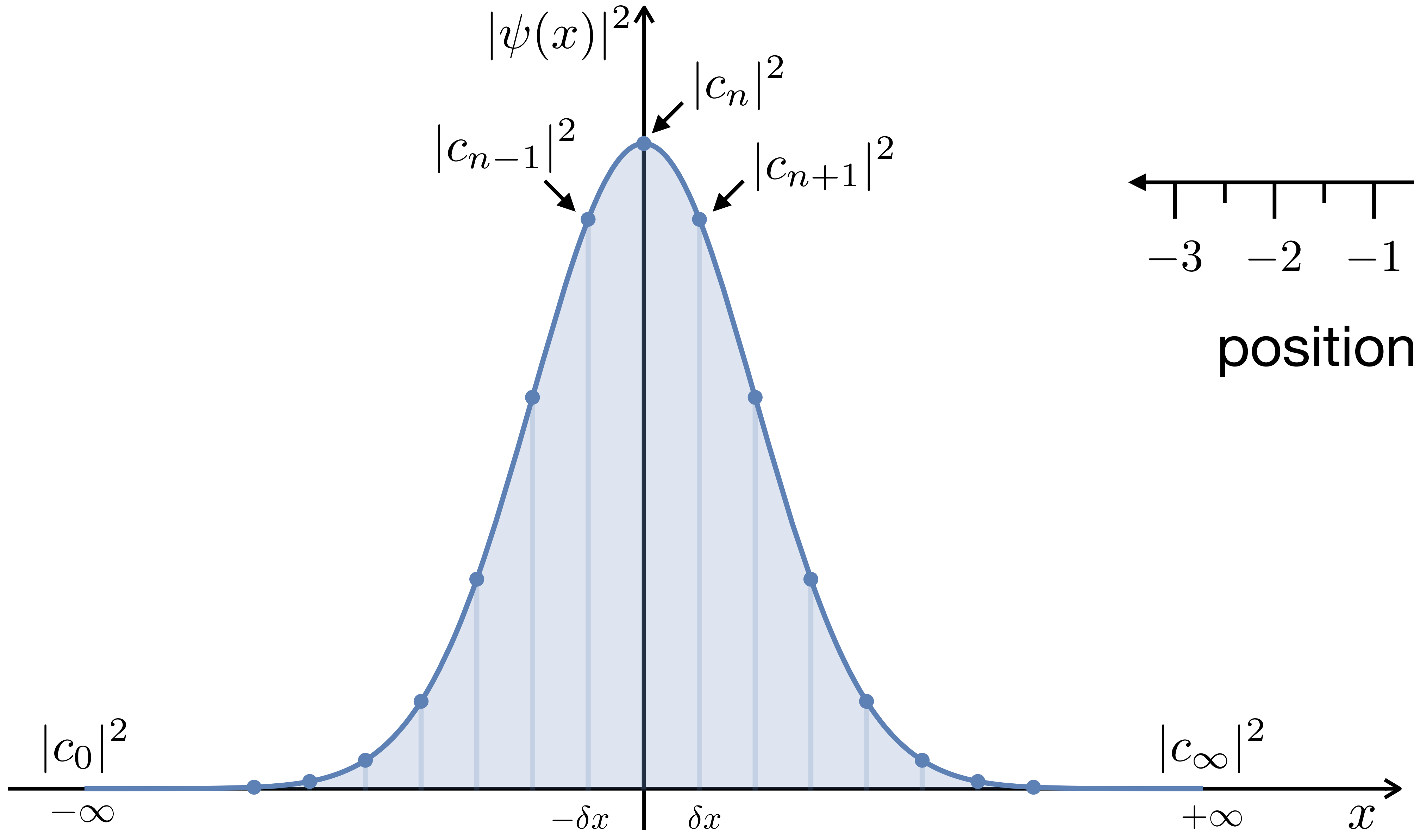
electron excitation

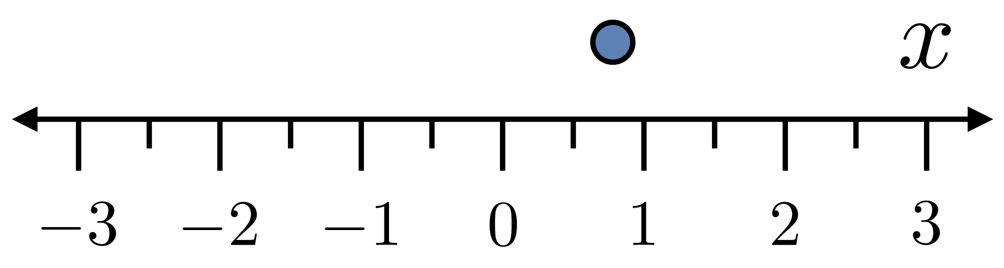


number of particles, e.g., photons

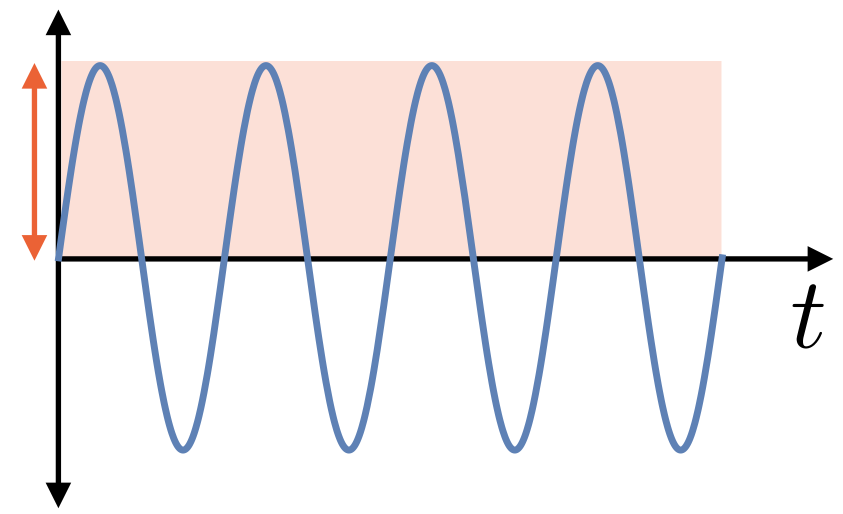
An infinite-dimensional state

$$|\psi\rangle = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \end{bmatrix} = \sum_{i=0}^{\infty} c_i |i\rangle$$





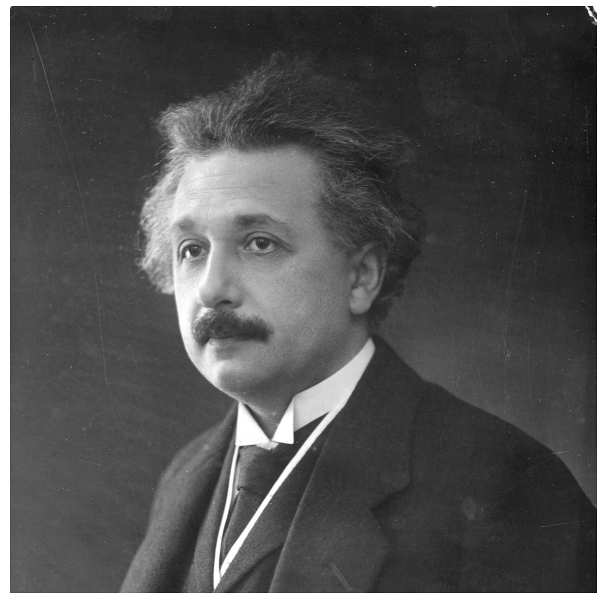
position of a particle



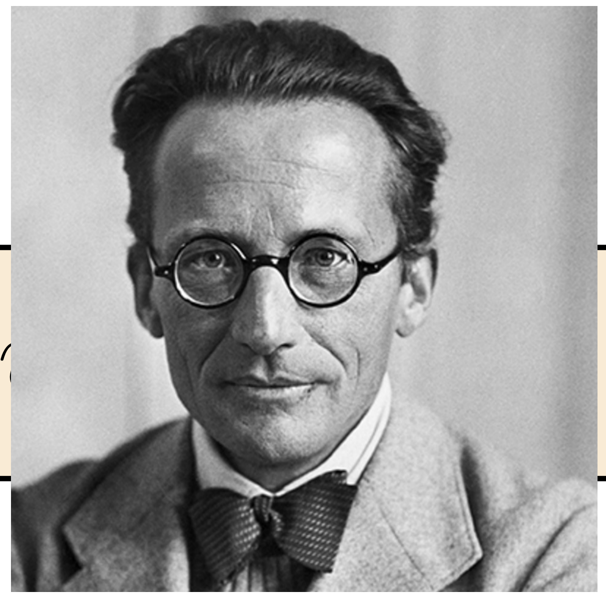
wave amplitude

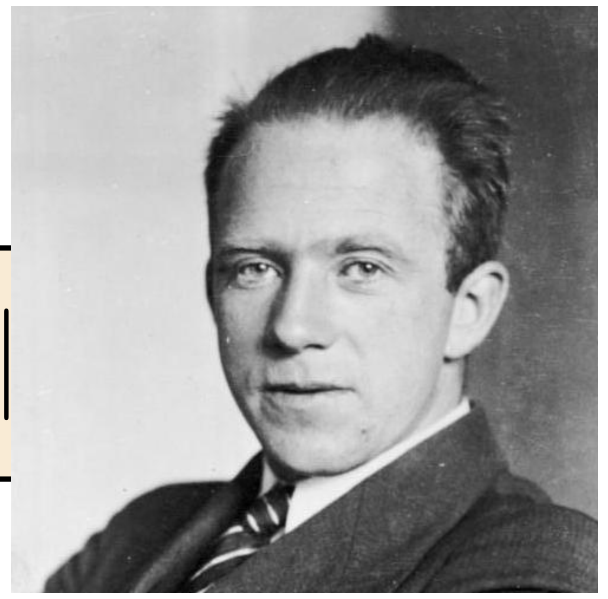
Quantum State Interpretation

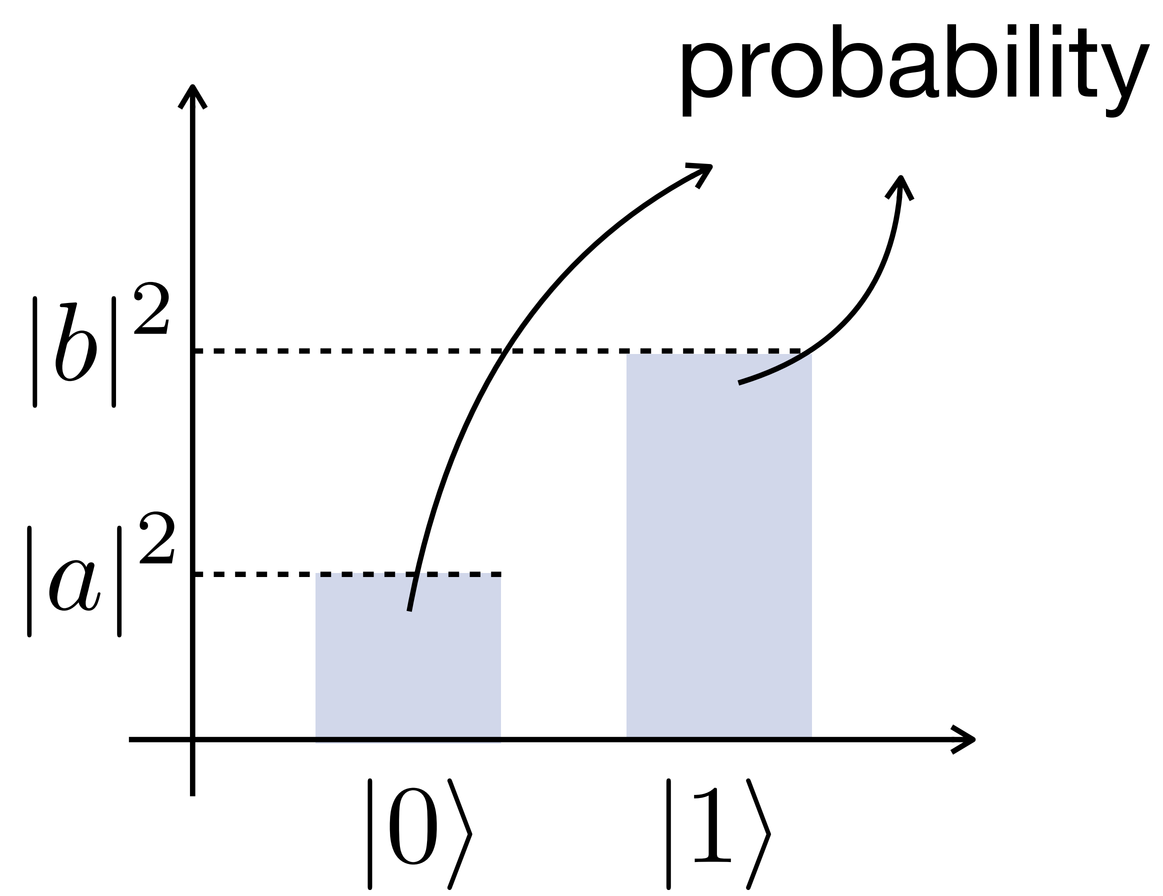

M. Born


A. Einstein


N. Bohr


E. Schrödinger


W. Heisenberg



Quantum State Interpretation	Probability Interpretation
A quantum state corresponds to a **statistical ensemble** of independent and identically prepared copies of a quantum system	**Frequentism**: The relative frequency of an event in the limit of sufficient many trials
A quantum state provides a complete description of an **individual** quantum system	**Bayesianism**: The degree of confidence of a hypothesis based on the prior knowledge

many worlds pilot waves Copenhagen interpretation quantum Bayesianism

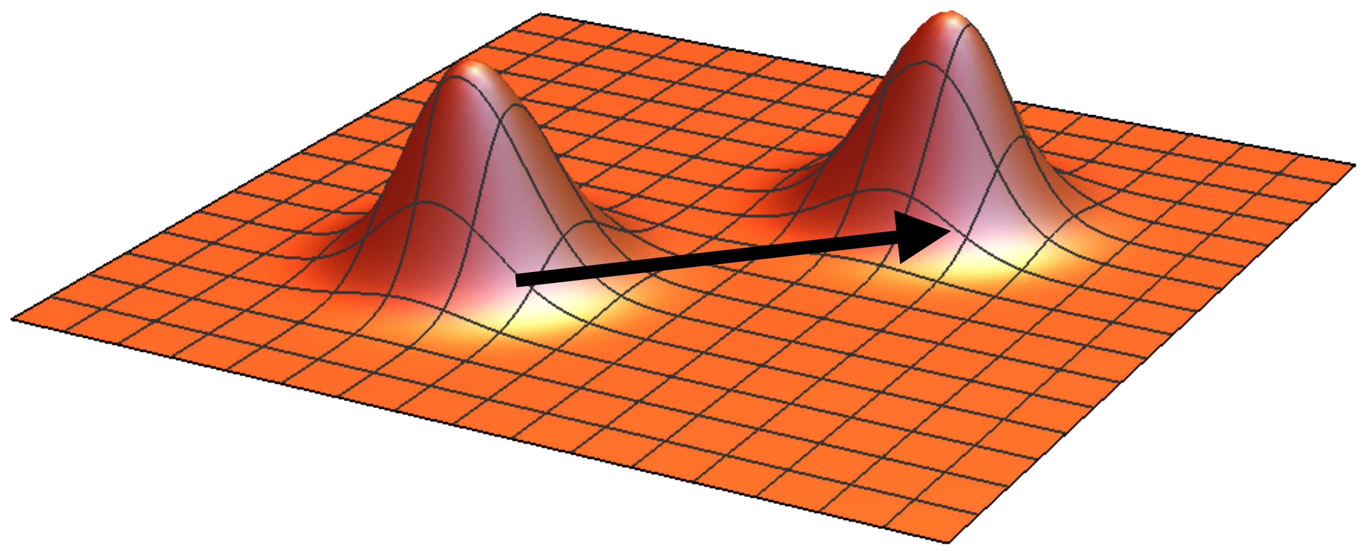
Lecture 1 — Introduction to Quantum Systems

- Introduction
  - When Classical Mechanics Fails
  - Review of Classical Systems
- Quantum Observables
- Quantum States
- **Evolution in Quantum Systems**
- Quantum Information

Evolution of a Quantum System

The **evolution** of a **classical property** is **deterministic** $\frac{\mathrm{d}x}{\mathrm{d}t} = y$

The **evolution** of a **quantum property** is **deterministic** $\frac{\mathrm{d}X}{\mathrm{d}t} = Y$



The **evolution** of a quantum state is described by a **unitary transformation** on the quantum state

$$|\psi\rangle \to U|\psi\rangle$$

- **Unitary** is a matrix that satisfies: $UU^\dagger = U^\dagger U = \mathbb{1}$ (preserves normalization)
- When $U = e^{\imath Ht/\hbar}$ where H is the Hamiltonian of the quantum system and $\hbar$ the reduced Planck constant the evolution is given by the **Schrödinger equation**

$$\imath\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = H|\psi\rangle$$

Example of Quantum Evolution

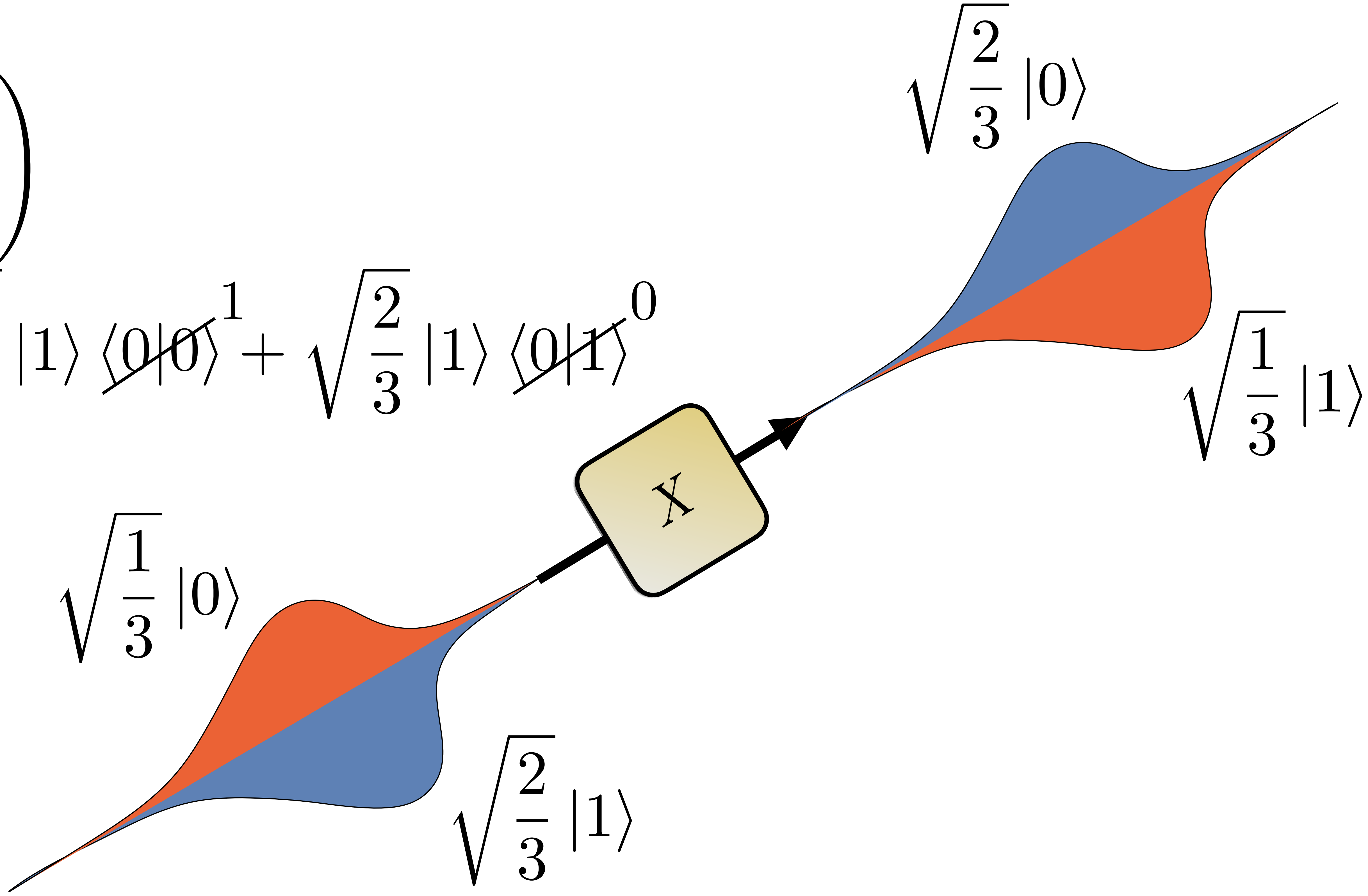
Let us have the quantum state $|\psi\rangle = \sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$

evolving according to the unitary matrix $X = |0\rangle\langle 1| + |1\rangle\langle 0|$

$$X|\psi\rangle = (|0\rangle\langle 1| + |1\rangle\langle 0|)\left(\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle\right)$$

$$= \sqrt{\frac{1}{3}}|0\rangle \cancel{\langle 1|0\rangle}^{0} + \sqrt{\frac{2}{3}}|0\rangle \cancel{\langle 1|1\rangle}^{1} + \sqrt{\frac{1}{3}}|1\rangle \cancel{\langle 0|0\rangle}^{1} + \sqrt{\frac{2}{3}}|1\rangle \cancel{\langle 0|1\rangle}^{0}$$

$$= \sqrt{\frac{2}{3}}|0\rangle + \sqrt{\frac{1}{3}}|1\rangle$$



$Z = \|0\rangle\langle 0\| - \|1\rangle\langle 1\|$	$Z\|\psi\rangle = ?$
$\mathrm{H} = \frac{1}{\sqrt{2}}(X + Z)$	$\mathrm{H}\|\psi\rangle = ?$

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Quantum Information

In **classical information theory** we encode information onto **bits**

bit: 0 or 1

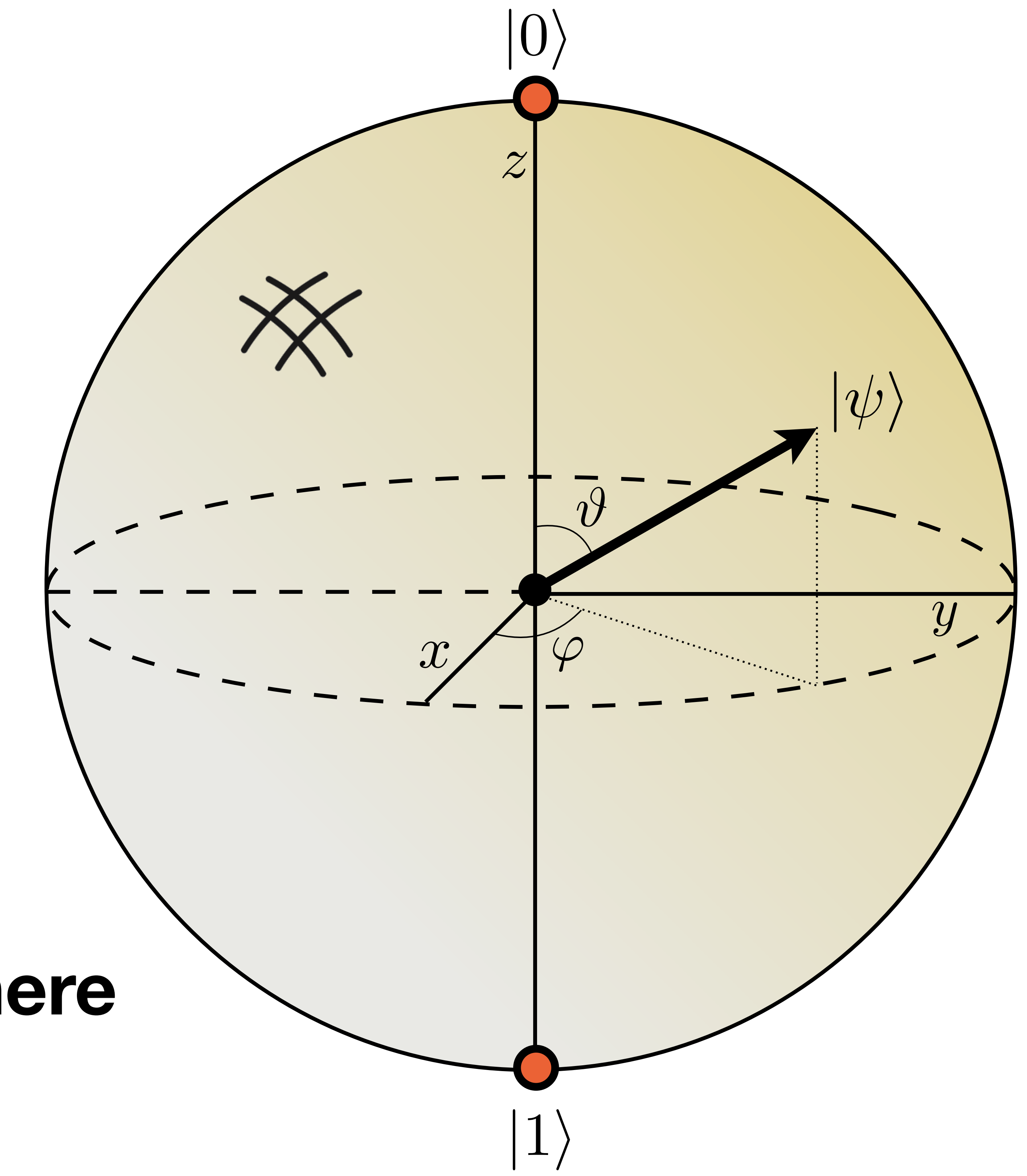
In **quantum information theory** we encode information onto **quantum bits** (qubits)

qubit: $a\left|0\right\rangle + b\left|1\right\rangle \quad \rightarrow \quad \left|0\right\rangle$ or $\left|1\right\rangle$

$$\left|\psi\right\rangle = a\left|0\right\rangle + b\left|1\right\rangle = e^{\imath\chi}\left(\cos\frac{\theta}{2}\left|0\right\rangle + e^{\imath\varphi}\sin\frac{\theta}{2}\left|1\right\rangle\right)$$

$$\chi \in [0, 2\pi]$$
$$\varphi \in [0, 2\pi]$$
$$\theta \in [0, \pi]$$

Qubit contains a bit as a special case



Bloch Sphere

Next Week

- **Lecture 1 — Introduction to Quantum Systems** (April 13, 2022)

- **Lecture 2 — Teleportation and Entanglement** (April 20, 2022)

- **Lecture 3 — Decoherence and Quantum Networks** (April 27, 2022)

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Suggested Bibliography

- Quantum Mechanics
  - J. Townsend - “A Modern Approach to Quantum Mechanics”
  - L. Ballentine - “Quantum Mechanics”
- Quantum Information
  - J. Audretsch - “Entangled Systems”
  - M. Nielsen, I. Chuang - “Quantum Computation and Quantum Information”

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